SUPPORTING INFORMATION

Triclinic metamaterials by tristable origami with reprogrammable frustration

Ke Liu,[1] Phanisri P. Pratapa,[2] Diego Misseroni,[3]
Tomohiro Tachi,[4] and Glaucio H. Paulino[5,*]

[1]*Department of Advanced Manufacturing and Robotics, Peking University, Beijing 100871, China*

[2]*Department of Civil Engineering, Indian Institute of Technology Madras, Chennai 600036, TN, India*

[3]*Department of Civil, Environmental and Mechanical Engineering, University of Trento, Trento 38123, Italy*

[4]*Graduate School of Arts and Sciences, University of Tokyo, Tokyo 153-8902, Japan*

[5]*Department of Civil and Environmental Engineering, Princeton Institute for the Science and Technology of Materials, Princeton University, Princeton, New Jersey, 08544, USA*

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gp1863@princeton.edu

I: Trimorph Geometry

The unit cell of the Trimorph pattern comprises a degree-4 vertex (denoted by O_5 in Fig. S1(a)) formed by the intersection of parallelogram shaped panels with edge lengths a and b and angles $+$, or , for $0 <$ 90 , 0 $<$. The sum of opposite panel angles is equal to 2 and therefore, the Trimorph vertex is at-foldable by virtue of Kawasaki theorem [1]. However, the vertex is not developable except when $= 90$ which makes the sum of panel angles 2. The con guration space or the folded state of the unit cell can be de ned by any of the dihedral or folding angles $_1$, $_2$, $_3$, $_4$, or (see Fig. S1(a)). These angles are all related to each other through the following equations obtained using the spherical law of cosines applied at the degree-4 vertex:

$$\cos \quad = \cos \quad \cos(\quad + \quad) + \sin \quad \sin(\quad + \quad) \cos \ _1 ; \tag{S1}$$

$$\cos \quad = \cos \quad \cos(\quad \quad) + \sin \quad \sin(\quad \quad) \cos \ _3 ; \tag{S2}$$

$$\cos \quad = \cos \quad \cos(\quad + \quad) + \sin \quad \sin(\quad + \quad) \cos \ _2 ; \tag{S3}$$

$$\cos \quad = \cos \quad \cos(\quad \quad) + \sin \quad \sin(\quad \quad) \cos \ _4 : \tag{S4}$$

By decomposing $_1 = \ ^0_1 \quad ^{00}_1$, we can further write the following equations using the spherical law of cosines:

$$\cos \quad = \cos(\quad + \quad) \cos \quad + \sin(\quad + \quad) \sin \quad \cos \ ^0_1 ; \tag{S5}$$

$$\cos(\quad \quad) = \cos \quad \cos \quad + \sin \quad \sin \quad \cos \ ^{00}_1 ; \tag{S6}$$

which leads to the following expression for $_1$:

$$_1 = \cos^{\ 1} \ \frac{\cos \quad \cos(\quad + \quad) \cos}{\sin(\quad + \quad) \sin} \quad \cos^{\ 1} \ \frac{\cos(\quad \quad) \quad \cos \ \cos}{\sin \quad \sin} \ : \tag{S7}$$

Note that here the sign ` ' depends on whether the crease corresponding to $_4$ is Mountain or Valley fold respectively (see Fig. S1(b),(c)). Finally, using the above equation in Eqn. S1 we obtain the following relation between and that allows us to study the con guration space:

$$\begin{aligned} \cos \quad &= \cos(\quad + \quad) \cos \quad + \sin(\quad + \quad) \sin \\ &\cos \ \cos^{\ 1} \ \frac{\cos \quad \cos(\quad + \quad) \cos}{\sin(\quad + \quad) \sin} \quad \cos^{\ 1} \ \frac{\cos(\quad \quad) \quad \cos \ \cos}{\sin \quad \sin} \end{aligned} \ : \tag{S8}$$

FIG. S1. (A) Geometry of Trimorph unit cell. (B) and (C) are spherical polygonal representations of the O_5 vertex in Eggbox mode and Miura Mode II respectively. (D) Con guration space of the Trimorph unit cell.

A: Con guration space

By simplifying Eqn. S8 using inverse trigonometric identities, we get:

$$\cos \;=\; \frac{(\cos \quad \cos\ )}{\sin^2}(2\cos^2 \qquad p\ \frac{}{(\cos(2 \qquad)\quad \cos\ )(\cos(2\ +\ )\quad \cos\ ))} + \frac{\cos\ \ \sin^2}{\sin^2}: \quad \text{(S9)}$$

Here the sign ` ' depends on whether the crease corresponding to is Mountain or Valley fold respectively. Further simpli cation of this equation leads us to a symmetric relation between and given by,

$$f(\ ;\ \ ) = 4\cos^2\ \ \cos^2 \qquad 4(\cos^2\ \ +\cos^2\ \ ) + C(\cos\ \ +\cos\ \ ) + D\cos\ \ \cos\ \ + E = 0\ ; \quad \text{(S10)}$$

where $C = 16\cos^2\ \ \cos\ $, $D = \ \ 8(\cos 2\ \ +\cos^2\ \ )$, $E = \ \ \sin^2 2 \qquad \cos^2\ \ (4 + 8\cos 2\ \ )$. It can be noted that $f(\ ;\ \ ) = f(\ ;\ \ )$. The symmetry is also evident from Fig. S1(d). This allows us to write

$$\cos \;=\; \frac{(\cos \quad \cos\ )}{\sin^2}(2\cos^2 \qquad p\ \frac{}{(\cos(2 \qquad)\quad \cos\ )(\cos(2\ +\ )\quad \cos\ ))} + \frac{\cos\ \ \sin^2}{\sin^2}: \quad \text{(S11)}$$

Here the sign ` ' depends on whether the crease corresponding to θ_2 is Mountain or Valley fold respectively.

B: Vertex coordinates

We consider a coordinate system with origin at vertex O_5 and x-axis along the fold line $\overrightarrow{O_2O_5}$. The xy-plane is assumed to coincide with panel $O_1O_2O_5O_4$ and z-axis is obtained by the right hand rule. The coordinates of the vertices can be obtained using the Rodrigues' rotation formula [2] for finite rotations:

$$\vec{v}_{rot} = \vec{v}\cos\theta + (\hat{\rho}\times\vec{v})\sin\theta + \hat{\rho}(\hat{\rho}\cdot\vec{v})(1-\cos\theta); \qquad \text{(S12)}$$

where $\vec{v}_{rot}$ is obtained by rotating a vector $\vec{v}$ about axis of rotation $\hat{\rho}$ by an angle θ using the right hand rule. The coordinates are obtained as:

$O_5 = (0, 0, 0)$

$O_2 = (-a, 0, 0)$

$O_1 = (-a + a\cos(\gamma_1 + \gamma_2),\ a\sin(\gamma_1 + \gamma_2), 0)$

$O_4 = (-a\cos(\gamma_1 + \gamma_2),\ a\sin(\gamma_1 + \gamma_2), 0)$

$O_6 = (-a\cos\gamma_1,\ a\cos\gamma_2\sin\gamma_1,\ a\sin\gamma_2\sin\gamma_1)$

$O_3 = (-a + a\cos\gamma_1,\ a\cos\gamma_2\sin\gamma_1,\ a\sin\gamma_2\sin\gamma_1)$

$O_8 = (-a(\cos(\gamma_1 + \gamma_2)\cos\gamma_1 + \sin(\gamma_1 + \gamma_2)\sin\gamma_2\cos\gamma_3),$
$a(\sin(\gamma_1 + \gamma_2)\cos\gamma_2\cos\gamma_3 - \cos(\gamma_1 + \gamma_2)\sin\gamma_3\cos\gamma_4),\ a(\sin\gamma_2\sin\gamma_3\sin\gamma_4))$

$O_7 = (-a\cos\gamma_1 - a(\cos(\gamma_1 + \gamma_2)\cos\gamma_1 + \sin(\gamma_1 + \gamma_2)\sin\gamma_2\cos\gamma_3),$
$a\sin(\gamma_1 + \gamma_2) - a(\sin(\gamma_1 + \gamma_2)\cos\gamma_2\cos\gamma_3 - \cos(\gamma_1 + \gamma_2)\sin\gamma_3\cos\gamma_4),\ a(\sin\gamma_2\sin\gamma_3\sin\gamma_4))$

$O_9 = (-a\cos\gamma_1 - a(\cos(\gamma_1 + \gamma_2)\cos\gamma_1 + \sin(\gamma_1 + \gamma_2)\sin\gamma_2\cos\gamma_3),$
$a\cos\gamma_2\sin\gamma_1 - a(\sin(\gamma_1 + \gamma_2)\cos\gamma_2\cos\gamma_3 - \cos(\gamma_1 + \gamma_2)\sin\gamma_3\cos\gamma_4),$
$a\sin\gamma_2\sin\gamma_3 - a(\sin\gamma_2\sin\gamma_3\sin\gamma_4))$

This coordinate framework allows us to obtain further analytical relations between the dihedral angles. For example, the condition that $\overrightarrow{O_5O_8}\cdot\overrightarrow{O_5O_6} = a^2\cos(\gamma_2 - \gamma_4)$ gives the

following relation between $_1$ and $_2$,

$$f(\ _1;\ _2) = \cos(\quad) + \cos(\ +\ )\cos^2\ + \sin(\ +\ )\sin\ \cos\ (\cos\ _1 + \cos\ _2)$$
$$\cos(\ +\ )\sin^2\ \cos\ _1\cos\ _2 + \sin^2\ \sin\ _1\sin\ _2 = 0\ ; \tag{S13}$$

which will be used in a later derivation. Similarly, by choosing a different coordinate system with origin at vertex O_5 and x-axis along the fold line $\overrightarrow{O_6O_5}$, we can use the condition that $\overrightarrow{O_5O_8}\ \overrightarrow{O_5O_4} = a^2\cos$ which gives the following relation between $_2$ and $_3$,

$$\cos\ + \cos(\ +\ )\cos(\quad)\cos$$
$$+\sin\ (\sin(\ +\ )\cos(\quad)\cos\ _2 + \cos(\ +\ )\sin(\quad)\cos\ _3)$$
$$\cos\ \sin(\quad)\sin(\ +\ )\cos\ _2\cos\ _3 + \sin(\quad)\sin(\ +\ )\sin\ _2\sin\ _3 = 0\ : \tag{S14}$$

C: Triclinic angles of the unit cell

FIG. S2. (A) Triclinic angles of the Trimorph unit cell. (B) Bounding box of the Trimorph unit cell.

The vertices O_1, O_3, O_7 and O_9 are co-planar and form a parallelogram with angle $_1$ at O_1 (see Fig. S2(a)). The angle $_1$ is calculated as follows:

$$\overrightarrow{O_1O_3}\ \overrightarrow{O_1O_7} = LW\cos\ _1 = 4a^2\sin\frac{}{2}\sin\frac{}{2}\cos\ _1\ : \tag{S15}$$

The dot product is calculated using the vertex coordinates obtained previously,

$$\overrightarrow{O_1O_3}\ \overrightarrow{O_1O_7}/(a^2) = \cos(\ +\ )\cos^2\ + \cos(\ +\ )\quad\cos(\ +\ )\sin^2\ \cos\ _1\cos\ _2$$
$$+\sin^2\ \sin\ _1\sin\ _2\quad 2\cos\ + \sin(\ +\ )\sin\ \cos\ (\cos\ _1 + \cos\ _2)$$
$$= \cos(\ +\ ) + \cos(\quad)\quad 2\cos\ :\ \text{(Using Eqn. S13)} \tag{S16}$$

Using this result in Eqn. S15, we get,

$$\cos\ _1 = \frac{\cos\ (\cos\ \ 1)}{2\sin\frac{}{2}\sin\frac{}{2}}: \tag{S17}$$

Equation S17 indicates that $_1$ depends on the folded state of the system, and is greater than 90 for > 0, making the base of the unit cell non-orthogonal.

The other two triclinic angles (see Fig. S2(a)) can be evaluated as

$$\cos\ _2 = \hat{r}\ \overset{!}{O_1O_7}/W\ ;$$
$$\cos\ _3 = \hat{r}\ \overset{!}{O_1O_3}/L\ ;$$

where $\hat{r} = [r_x; r_y; r_z]$ is a unit vector along one of the edges $\overset{!}{O_1O_1}$ of the bounding box of the unit cell as shown in Fig. S2(b). The vector $\hat{r}$ can be obtained by solving the following set of equations.

$$\hat{r}\ (\overset{!}{O_2O_1}\ \ \overset{!}{O_2O_3}) = 0\ ;$$
$$\hat{r}\ (\overset{!}{O_4O_7}\ \ \overset{!}{O_4O_1}) = 0\ ;$$
$$r_x^2 + r_y^2 + r_z^2 = 1\ :$$

II: In-plane stretching of Trimorph

Figure S3 provides a schematic of the deformation of the base of the unit cell under in-plane stretching of the tessellation. Stretching in one of the $W = \overset{!}{O_1O_7}$ or $L = \overset{!}{O_1O_3}$ directions results in a deformation in the other direction as well. The inter-dependence of deformations in both these lattice directions can be calculated by looking at the corresponding changes in and . Using $f(\ ;\ ) = 0$ from Eqn. S10, we have,

$$df = \frac{@f}{@}d\ + \frac{@f}{@}d\ = 0\ =) \quad \frac{d}{d} = \quad \frac{@f}{@}\ \frac{@f}{@}\ ^1: \tag{S18}$$

To characterize the deformations of the system during in-plane stretching, we study the ratio of strains: (1) along the lattice directions and (2) along the principal directions. The latter corresponds to the conventional notion of Poisson's ratio where there is no shear strain. However, the former quantity provides a more convenient notion that allows us to gain insight into the in-plane mechanics of the lattice.

FIG. S3. In-plane stretching of Trimorph unit cell. (a) Extension considered along the W direction. (b) 2D schematic of the triclinic tessellation and the lattice vectors W and L. (c) Shape of a unit cell base before and after the deformation due to in-plane stretching.

A: Lattice Poisson's Ratio (LPR) in stretching

We de ne Lattice Poisson's Ratio (LPR) in stretching as the ratio of strains along the lattice directions $\varepsilon_L = dL/L$ and $\varepsilon_W = dW/W$, given by,

$$\nu_{stretch} = \frac{\varepsilon_L}{\varepsilon_W} = \frac{W}{L}\frac{dL}{d}\frac{d}{dW}\frac{d}{d}$$

$$L^2 = 4a^2\sin^2\frac{}{2} \Rightarrow \frac{dL}{d} = \frac{a^2\sin}{L}$$

$$W^2 = 4a^2\sin^2\frac{}{2} \Rightarrow \frac{dW}{d} = \frac{a^2\sin}{W}$$

$$\Rightarrow \nu_{stretch} = \frac{\tan(\ =2)}{\tan(\ =2)}\frac{d}{d} :$$

Using the derivative from Eqn. S18, we get

$$\nu_{stretch} = \frac{\sin^2(\ =2)}{\sin^2(\ =2)}\frac{(\cos 2\ +\cos^2\ )\cos\ \ 2\cos^2\ \cos\ +\sin^2\ \cos}{(\cos 2\ +\cos^2\ )\cos\ \ 2\cos^2\ \cos\ +\sin^2\ \cos} : \qquad \text{(S19)}$$

We denote $\nu_{stretch}$ as ν_{WL}.

B: Principal Poisson's Ratio (PPR) in stretching

The deformation gradient describing a linear deformation shown in Fig. S3(c) can be obtained as,

$$F = \begin{bmatrix} (1+\frac{dW}{W}) & (\frac{dL}{L} - \frac{dW}{W})\frac{\cos\ _1}{\sin\ _1} - d\ _1 \\ 0 & 1+\frac{dL}{L} + d\ _1\frac{\cos\ _1}{\sin\ _1} \end{bmatrix};$$

The in nitesimal strain tensor is calculated as,

$$E = \frac{1}{2}(F^T + F) \quad I$$

$$= \begin{bmatrix} \frac{dW}{W} & \frac{1}{2}(\frac{dL}{L} - \frac{dW}{W})\frac{\cos\ _1}{\sin\ _1} - \frac{d\ _1}{2} \\ \frac{1}{2}(\frac{dL}{L} - \frac{dW}{W})\frac{\cos\ _1}{\sin\ _1} - \frac{d\ _1}{2} & \frac{dL}{L} + d\ _1\frac{\cos\ _1}{\sin\ _1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{dW}{d}\frac{1}{W} & \frac{1}{2}(\frac{dL}{d}\frac{d}{d}\frac{1}{L} - \frac{dW}{d}\frac{1}{W})\frac{\cos\ _1}{\sin\ _1} - \frac{d\ _1}{d}\frac{1}{2} \\ \frac{1}{2}(\frac{dL}{d}\frac{d}{d}\frac{1}{L} - \frac{dW}{d}\frac{1}{W})\frac{\cos\ _1}{\sin\ _1} - \frac{d\ _1}{d}\frac{1}{2} & \frac{dL}{d}\frac{d}{d}\frac{1}{L} + \frac{d\ _1}{d}\frac{\cos\ _1}{\sin\ _1} \end{bmatrix} d\ : \quad \text{(S20)}$$

We de ne the Principal Poisson's Ratio (PPR) in stretching ($^p_{\text{stretch}}$) as the ratio of principal strains calculated as the ratio of the eigenvalues of the above strain tensor at any given folded state of the system.

C: Shear-Normal coupling coe cient

The expression for the strain tensor in Eqn. S20 can be further simpli ed and expressed in terms of LPR ($_{WL}$) as follows

$$E = \begin{bmatrix} 1 & \cot\ _1 \\ \cot\ _1\ (1 - _{WL})\cot^2\ _1 - _{WL} & \end{bmatrix} "_W d\ : \quad \text{(S21)}$$

From this, the shear-normal coupling coe cient, de ned as the negative ratio of shear strain to the applied normal strain, is given by

$$= \frac{2"_{WL}}{"_W} = 2\cot\ _1 : \quad \text{(S22)}$$

D: Load vs. deformation of Trimorph pattern

The elastic energy of a Trimorph pattern with N_W and N_L number of cells along the W and L directions, respectively, at a folded state is given by

$$U(\ ) = N_W(2N_L - 1)(U_1 + U_3) + N_L(2N_W - 1)(U_2 + U_4)\ ; \quad \text{(S23)}$$

where $U_i = U_i(\gamma_i(\theta); \rho_i; K_F)$ (for $i \in \{1,2,3,4\}$) is the energy stored in crease with dihedral angle γ_i. Here ρ_i are the rest angles and K_F is the crease rotational stiffness. The constitutive relation for the crease given by $M(\gamma_i) = dU_i/d\gamma_i$ can be adopted from nonlinear mechanics of origami [3].

The in-plane uniaxial load required to deform the pattern along the W direction is given by

$$\frac{dU(\theta)}{dW} = \frac{dU}{d\theta}\frac{d\theta}{dW} = \frac{N_W(2N_L - 1)}{a\cos(\theta/2)}\left(\frac{dU_1}{d\theta} + \frac{dU_3}{d\theta}\right) + \frac{N_L(2N_W - 1)}{a\cos(\theta/2)}\left(\frac{dU_2}{d\theta} + \frac{dU_4}{d\theta}\right). \quad \text{(S24)}$$

The above equation can be re-written as

$$\frac{dU(\theta)}{dW} = \frac{N_W(2N_L - 1)}{a\cos(\theta/2)}\left(M(\gamma_1)\frac{d\gamma_1}{d\theta} + M(\gamma_3)\frac{d\gamma_3}{d\theta}\right) + \frac{N_L(2N_W - 1)}{a\cos(\theta/2)}\left(M(\gamma_2)\frac{d\gamma_2}{d\theta} + M(\gamma_4)\frac{d\gamma_4}{d\theta}\right); \quad \text{(S25)}$$

where

$$\frac{d\gamma_1}{d\theta} = \frac{\sin\alpha}{\sin\beta\,\sin(\alpha+\beta)\sin\gamma_1}; \quad \text{(S26)}$$

$$\frac{d\gamma_2}{d\theta} = \frac{\sin\alpha}{\sin\beta\,\sin(\alpha+\beta)\sin\gamma_2}\frac{d\rho}{d\theta}; \quad \text{(S27)}$$

$$\frac{d\gamma_3}{d\theta} = \frac{\sin\alpha}{\sin\beta\,\sin(\beta-\alpha)\sin\gamma_3}; \quad \text{(S28)}$$

$$\frac{d\gamma_4}{d\theta} = \frac{\sin\alpha}{\sin\beta\,\sin(\beta-\alpha)\sin\gamma_4}\frac{d\rho}{d\theta}; \quad \text{(S29)}$$

In the above equations, $d\rho/d\theta$ can be evaluated using Eqn. S18.

III: Out-of-plane bending of Trimorph

The bending of the origami tessellation can be modeled by allowing the panels to undergo deformation. Previous research has found that imposing infinitesimal rotations ω_i ($i = 1,2,3,4$) across shorter diagonals of the panels (see Fig. S4(a)) is sufficient to capture first order deformation to define the discrete curvatures of the tessellation [4, 5]. Analogous to in-plane stretching, one can characterize out-of-plane bending deformation through the negative of the ratio of curvatures along either the lattice or the principal directions [6]. Accordingly, we define the Lattice and Principal Poisson's Ratios (LPR and PPR) for bending as the aforementioned ratio of curvatures along corresponding directions.

A: Vertex coordinates after bending

The coordinates of the vertices after application of the in nitesimal rotations are obtained using Rodrigues' rotation formula [2] for small rotations as:

$$O_5^0 = O_5 = (0\,;0;0)$$

$$O_2^0 = O_2 = (\ \ a;0;0)$$

$$O_4^0 = O_4 = (\ \ a\cos(\ \ +\ \ );\ \ a\sin(\ \ +\ \ );0)$$

$$O_6^0 = O_6 = (\ \ a\cos\,;\ \ \ a\cos\ {}_2\sin\ ;\ \ \ a\sin\ {}_2\sin\ )$$

$$O_1^0 = (\ \ a\ \ \ a\cos(\ \ +\ \ );\ \ a\sin(\ \ +\ \ );a^2\sin(\ \ +\ \ )\frac{!\ {}_1}{\grave{}\ {}_1})$$

$$O_3^0 = (\ \ a\ \ \ a\cos\,;\ \ \ a\cos\ {}_2\sin\ \ +(a^2\sin\ {}_2\sin\ )\frac{!\ {}_2}{\grave{}\ {}_2};\ \ a\sin\ {}_2\sin\ \ \ \ (a^2\cos\ {}_2\sin\ )\frac{!\ {}_2}{\grave{}\ {}_2})\qquad ;$$

where the lengths of the panel diagonals $!O_2O_4$, $!O_2O_6$, $!O_6O_8$, $!O_4O_8$ are denoted as $\grave{}_1$, $\grave{}_2$, $\grave{}_3$ and $\grave{}_4$ respectively.

FIG. S4. Out-of-plane bending of Trimorph unit cell. (A) Panel bending through in nitesimal rotations about the diagonals. (B) and (C) Triangular faces along the W direction before and after bending. (D) Parallelogram shaped base of the Trimorph unit cell and its perturbed shape due to a single panel bending.

B: Constraints on bending rotations

The in nitesimal rotations on the four panels of the unit cell add four additional degrees of freedom (DOF) and have to be constrained in an appropriate way to simulate the bending mode of deformation. We impose two conditions that reduce the DOF leading to a unique bending mode: (i) Tessellation Boundary Condition (TBC), and (ii) Frame constraint.

1: Tessellation Boundary Condition

This constraint ensures that the periodic boundary conditions on the unit cell are still satis ed after bending deformations are applied. For this, we impose $\angle O_1^0O_2^0O_3^0 = \angle O_7^0O_8^0O_9^0$ (see Fig. S4(b),(c)) and $\angle O_1^0O_4^0O_7^0 = \angle O_3^0O_6^0O_9^0$. This translates as $\vec{O_2O_3}\,\vec{O_2O_1} = \vec{O_8O_9}\,\vec{O_8O_7}$ and $\vec{O_4O_1}\,\vec{O_4O_7} = \vec{O_6O_9}\,\vec{O_6O_3}$ respectively. Since, the angles are independent of the choice of coordinate system we can choose the axis conveniently within the plane of each of the four panels while we calculate the corresponding dot products mentioned above. Evaluating the above conditions we arrive at the following constraints for the in nitesimal rotations,

$$\sin(\quad + \quad)\sin\quad\sin\ {}_2\,\frac{!_2}{\hat{}_2} + \frac{!_1}{\hat{}_1} = \sin(\quad\quad)\sin\quad\sin\ {}_4\,\frac{!_3}{\hat{}_3} + \frac{!_4}{\hat{}_4}\ ; \qquad \text{(S30)}$$

$$\sin(\quad + \quad)\sin\quad\sin\ {}_1\,\frac{!_4}{\hat{}_4} + \frac{!_1}{\hat{}_1} = \sin(\quad\quad)\sin\quad\sin\ {}_3\,\frac{!_3}{\hat{}_3} + \frac{!_2}{\hat{}_2}\ : \qquad \text{(S31)}$$

2: Frame constraint

We want to ensure that the frame (local co-ordinate system) in the bent con guration is unchanged. For this, the base of the parallelogram $O_1O_3O_9O_7$ should not change in shape (no shear or twist) so as to give a bending deformation mode. We de ne $\angle O_3O_1O_7 = {}_{1;I}$, $\angle O_7O_9O_3 = {}_{1;III}$, $\angle O_1O_7O_9 = {}_{1;IV}$ and $\angle O_1O_3O_9 = {}_{1;II}$ before applying the in nitesimal rotations, where, ${}_{1;I} = {}_{1;III} = \quad {}_{1;II} = \quad {}_{1;IV} = {}_1$. In nitesimal rotation $!_1$ takes O_1 to O_1^0 and the corresponding angle between the mid lines ${}_{2;I}$ becomes ${}^0_{1;I}$ (see Fig. S4(d)). After all the rotations are applied, the parallelogram angle (between the mid lines $\vec{O_{17}O_{39}}$ and $\vec{O_{13}O_{79}}$) changes from ${}_1$ to ${}^0_1$. (Note that O_{17} is obtained as mid point of O_1 and O_7.)

The angle ${}^0_1$ of the unit cell base in the bent con guration depends on the positions of

the vertices O_1^0, O_3^0, O_9^0 and O_7^0 which should satisfy the following condition,

$$\overset{0}{1} \quad _1 = \quad _1(!\ _1;!\ _2;!\ _3;!\ _4) = \quad _1(!\ _1) + \quad _1(!\ _2) + \quad _1(!\ _3) + \quad _1(!\ _4) = 0 ;$$

where $_1(!\ _i) = (\cos \overset{0}{}_{1;i} \quad \cos \ _{1;i})=\sin \ _i$, for $i = I;II;III;IV$. Since, $\sin \ _{1;I} = \sin \ _{1;II} = \sin \ _{1;III} = \sin \ _{1;IV} = \sin \ _1$, the constraint becomes:

$$(\cos \overset{0}{}_{1;I} \quad \cos \ _{1;I}) + (\cos \overset{0}{}_{1;II} \quad \cos \ _{1;II}) + (\cos \overset{0}{}_{1;III} \quad \cos \ _{1;III}) + (\cos \overset{0}{}_{1;IV} \quad \cos \ _{1;IV}) = 0 :$$

This can be calculated as:

$$(\dot{O}_{1^07}O_{39} \quad \dot{O}_{1^03}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79}) + (\dot{O}_{17}O_{3^09} \quad \dot{O}_{13^0}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79})$$
$$+(\dot{O}_{17^0}O_{39} \quad \dot{O}_{13}O_{7^09} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79}) + (\dot{O}_{17}O_{39^0} \quad \dot{O}_{13}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79^0}) = 0 :$$

Since, the constraint is just a summation of change in angles, each of the above terms can be calculated by using di erent coordinate system that will allow simple expressions as given below:

$$\dot{O}_{1^07}O_{39} \quad \dot{O}_{1^03}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79} = \sin(\quad + \quad) \sin \quad (\sin \ _1 + \sin \ _2)\frac{!\ _1}{\ }(a^3=2)$$
$$\dot{O}_{17}O_{3^09} \quad \dot{O}_{13^0}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79} = \quad \sin \quad (\sin(\quad + \quad) \sin \ _2 + \sin(\quad \quad) \sin \ _3)\frac{!\ _2}{\ }(a^3=2)$$
$$\dot{O}_{17}O_{39^0} \quad \dot{O}_{13}O_{79} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79^0} = \sin(\quad \quad) \sin \quad (\sin \ _3 + \sin \ _4)\frac{!\ _3}{\ }(a^3=2)$$
$$\dot{O}_{17^0}O_{39} \quad \dot{O}_{13}O_{7^09} \quad \dot{O}_{17}O_{39} \quad \dot{O}_{13}O_{79} = \quad \sin \quad (\sin(\quad \quad) \sin \ _4 + \sin(\quad + \quad) \sin \ _1)\frac{!\ _4}{\ }(a^3=2)$$

Finally, the frame constraint is given by:

$$\sin(\quad + \quad) \sin \quad (\sin \ _1 + \sin \ _2)\frac{!\ _1}{\ } \quad \sin \quad (\sin(\quad + \quad) \sin \ _2 + \sin(\quad \quad) \sin \ _3)\frac{!\ _2}{\ }$$
$$+\sin(\quad \quad) \sin \quad (\sin \ _3 + \sin \ _4)\frac{!\ _3}{\ } \quad \sin \quad (\sin(\quad \quad) \sin \ _4 + \sin(\quad + \quad) \sin \ _1)\frac{!\ _4}{\ } = 0 : \tag{S32}$$

3: Relation between rotations

Solving the three constraints (Eqns. S30, S31, S32), we arrive at the following relations between the rotations that satisfy the constraints.

$$\frac{\omega_2}{\dot{\gamma}_2} = \frac{\sin(\alpha_1 + \alpha_2)\sin\alpha_1}{\sin(\alpha_3)\sin\alpha_3}\frac{\omega_1}{\dot{\gamma}_1}$$
$$\frac{\omega_4}{\dot{\gamma}_4} = \frac{\sin(\alpha_1 + \alpha_2)\sin\alpha_2}{\sin(\alpha_3)\sin\alpha_4}\frac{\omega_1}{\dot{\gamma}_1}$$
$$\frac{\omega_3}{\dot{\gamma}_3} = \frac{\sin^2(\alpha_1 + \alpha_2)\sin\alpha_2\sin\alpha_1}{\sin^2(\alpha_3)\sin\alpha_4\sin\alpha_3}\frac{\omega_1}{\dot{\gamma}_1} \qquad \text{(S33)}$$

C: Bending curvatures

We characterize the bent conguration of a tessellation through a discrete description of space curves along the lattice directions W and L embedded in a parameterized surface. The normal curvature of this discrete surface formed by the bent tessellation can be evaluated through the second fundamental form at a reference unit cell about which the bending is concentrated. Within the $\{\hat{W}, \hat{L}\}$ basis, the normal curvature along any lattice direction is calculated as the normal component of the change in the lattice vector per unit length. Here, "^" represents the unit normalized vectors. We denote M_{123}, M_{789}, M_{147} and M_{369} as the medians (which are also the altitudes) of the triangles $O_1O_2O_3$, $O_7O_8O_9$, $O_1O_4O_7$ and $O_3O_6O_9$ respectively (see Fig. S4(c)). As the unit cell undergoes bending, M_{123} rotates to M'_{123} by a small angle ω^0_W about an axes $\hat{p}^0$, and M_{789} rotates to M'_{789} by a small angle ω^{00}_W about an axes $\hat{p}^{00}$. Correspondingly, W rotates to W' and W'' respectively as shown in Fig. S4(c). Therefore, we can write

$$\hat{W}' - \hat{W} = \omega^0_W \hat{p}^0 \times \hat{W} = (\hat{M}'_{123} - \hat{M}_{123}) \times \hat{W}; \qquad \text{(S34)}$$
$$\hat{W}'' - \hat{W}' = \omega^{00}_W \hat{p}^{00} \times \hat{W}' = (\hat{M}'_{789} - \hat{M}_{789}) \times \hat{W}'. \qquad \text{(S35)}$$

Combining these two equations, we get

$$\hat{W}'' - \hat{W} = \left[(\hat{M}'_{123} - \hat{M}_{123}) \times (\hat{M}'_{789} - \hat{M}_{789})\right] \times \hat{W}. \qquad \text{(S36)}$$

Using a similar description, we can also get

$$\hat{L}'' - \hat{L} = \left[(\hat{M}'_{147} - \hat{M}_{147}) \times (\hat{M}'_{369} - \hat{M}_{369})\right] \times \hat{W}. \qquad \text{(S37)}$$

The normal curvatures along W and L directions can then be defined as,

$$\kappa_W = \frac{(W'' \times W') \cdot \hat{n}}{W^2}; \tag{S38}$$

$$\kappa_L = \frac{(L'' \times L') \cdot \hat{n}}{L^2}; \tag{S39}$$

where $\hat{n} = (W \times L)/|W \times L|$. Therefore, the curvatures can be calculated as,

$$\kappa_W = \frac{\left[(\hat{M}'_{123} \times \hat{M}_{123}) \times \hat{W} \quad (\hat{M}'_{789} \times \hat{M}_{789}) \times \hat{W}\right] \cdot \hat{n}}{W}; \tag{S40}$$

$$\kappa_L = \frac{\left[(\hat{M}'_{147} \times \hat{M}_{147}) \times \hat{L} \quad (\hat{M}'_{369} \times \hat{M}_{369}) \times \hat{L}\right] \cdot \hat{n}}{L}. \tag{S41}$$

However, evaluating the above expressions analytically becomes very cumbersome using a single coordinate system for all the vertices. Hence, we re-write the above equations so that each of the scalar terms would be independent of the choice of coordinate system and can be calculated by suitably modifying the axes as discussed previously. Finally, we have the following equations to calculate the curvatures,

$$\kappa_W = \frac{[(M'_{123} \times M_{123}) \cdot W_{123}]\ (W_{123} \times L_{123}) \quad [(M'_{789} \times M_{789}) \cdot W_{789}]\ (W_{789} \times L_{789})}{8a^5 \sin^2\frac{}{2} \sin\frac{}{2} \cos^2\frac{}{2} \cos\ _1};$$

$$\kappa_L = \frac{[(M'_{147} \times M_{147}) \cdot L_{147}]\ (W_{147} \times L_{147}) \quad [(M'_{369} \times M_{369}) \cdot L_{369}]\ (W_{369} \times L_{369})}{8a^5 \sin^2\frac{}{2} \sin\frac{}{2} \cos^2\frac{}{2} \cos\ _1};$$

where

$$W_{123} = [a(1 \quad \cos\ ); \quad a(\cos\ \sin(\ +\ ) \quad \cos\ _1 \cos(\ +\ ) \sin\ ); \quad a \sin\ \sin\ _1]$$

$$L_{123} = [a(\cos(\ +\ ) \quad \cos\ ); a(\sin(\ +\ ) \quad \cos\ _2 \sin\ ); \quad a \sin\ \sin\ _2]$$

$$\frac{(M'_{123} \times M_{123})}{(\frac{a^3}{4})} = [\frac{!_1}{`_1} \sin(\ +\ )(\sin(\ +\ ) + \cos\ _2 \sin\ ) \quad \frac{!_2}{`_2} \sin\ (\sin\ + \cos\ _2 \sin(\ +\ ));$$
$$\frac{!_1}{`_1} \sin(\ +\ )(\cos(\ +\ ) + \cos\ ) + \frac{!_2}{`_2} \sin\ \cos\ _2(\cos(\ +\ ) + \cos\ );$$
$$\frac{!_2}{`_2} \sin\ \sin\ _2(\cos(\ +\ ) + \cos\ )]$$

$$W_{789} = \quad [a(1 \quad \cos\ ); \quad a(\cos\ \sin(\quad) \quad \cos\ _3 \cos(\quad) \sin\ ); \quad a \sin\ \sin\ _3]$$

$$L_{789} = \quad [a(\cos(\quad) \quad \cos\ ); a(\sin(\quad) \quad \cos\ _4 \sin\ ); \quad a \sin\ \sin\ _4]$$

$$\frac{(M'_{789} \times M_{789})}{(\frac{a^3}{4})} = [\frac{!_3}{`_3} \sin(\quad)(\sin(\quad) + \cos\ _4 \sin\ ) \quad \frac{!_4}{`_4} \sin\ (\sin\ + \cos\ _4 \sin(\quad));$$
$$\frac{!_3}{`_3} \sin(\quad)(\cos(\quad) + \cos\ ) + \frac{!_4}{`_4} \sin\ \cos\ _4(\cos(\quad) + \cos\ );$$
$$\frac{!_4}{`_4} \sin\ \sin\ _4(\cos(\quad) + \cos\ )]$$

$$L_{143} = [\,a(1 \quad \cos\ );\ \ a(\cos(\quad)\sin \quad \cos\ {}_4\cos\ \sin(\quad));\ \ a\sin(\quad)\sin\ {}_4]$$

$$W_{143} = \ [a(\cos \quad \cos(\ +\ ));a(\sin \quad \cos\ {}_1\sin(\ +\ ));\ \ a\sin(\ +\ )\sin\ {}_1]$$

$$\frac{(M^{\,0}_{143} \quad M_{143})}{(\frac{a^3}{4})} = [\,\frac{!_4}{\dot{}_4}\sin\ (\sin\ + \cos\ {}_1\sin(\ +\ ))\quad \frac{!_1}{\dot{}_1}\sin(\ +\ )(\sin(\ +\ ) + \cos\ {}_1\sin\ );$$
$$\frac{!_4}{\dot{}_4}\sin\ (\cos\ + \cos(\ +\ )) + \frac{!_1}{\dot{}_1}\sin(\ +\ )\cos\ {}_1(\cos\ + \cos(\ +\ ));$$
$$\frac{!_1}{\dot{}_1}\sin(\ +\ )\sin\ {}_1(\cos\ + \cos(\ +\ ))]$$

$$L_{369} = [\ \ a(1 \quad \cos\ );a(\cos(\ +\ )\sin \quad \cos\ {}_2\cos\ \sin(\ +\ ));a\sin(\ +\ )\sin\ {}_2]$$

$$W_{369} = [\,a(\cos \quad \cos(\quad));a(\sin \quad \cos\ {}_3\sin(\quad));\ \ a\sin(\quad)\sin\ {}_3]$$

$$\frac{(M^{\,0}_{369} \quad M_{369})}{(\frac{a^3}{4})} = [\,\frac{!_2}{\dot{}_2}\sin\ (\sin\ + \cos\ {}_3\sin(\quad))\quad \frac{!_3}{\dot{}_3}\sin(\quad)(\sin(\quad) + \cos\ {}_3\sin\ );$$
$$\frac{!_2}{\dot{}_2}\sin\ (\cos\ + \cos(\quad)) + \frac{!_3}{\dot{}_3}\sin(\quad)\cos\ {}_3(\cos\ + \cos(\quad));$$
$$\frac{!_3}{\dot{}_3}\sin(\quad)\sin\ {}_3(\cos\ + \cos(\quad))]$$

The in nitesimal rotations $!_i$ in the above expressions can all be reduced in terms of $!_1$ by using Eqns. S33. We shall characterize the bending response of the Trimorph system through the ratio of curvatures along (i) lattice directions and (ii) principal directions.

D: Lattice Poisson's Ratio (LPR) in bending

We de ne LPR in bending as the ratio of curvatures along the tessellation directions:

$$\dot{}_{\text{bend}} = \quad \frac{L}{W};$$

which can be analytically evaluated as

$$\dot{}_{\text{bend}} = \frac{\sin\ \frac{}{2}\cos^2\frac{}{2}}{\sin\ \frac{}{2}\cos^2\frac{}{2}}\ \frac{[(M^{\,0}_{147} \quad M_{147}) \quad L_{147}]\ (W_{147} \quad L_{147})}{[(M^{\,0}_{123} \quad M_{123}) \quad W_{123}]\ (W_{123} \quad L_{123})}\ \frac{[(M^{\,0}_{369} \quad M_{369}) \quad L_{369}]\ (W_{369} \quad L_{369})}{[(M^{\,0}_{789} \quad M_{789}) \quad W_{789}]\ (W_{789} \quad L_{789})}\ : \tag{S42}$$

E: Principal Poisson's Ratio (PPR) in bending

In order to calculate the principal curvatures, we need to look at the matrix representation of the di erential of the Gauss map [7] within the $\{\hat{W}, \hat{L}\}$ basis given by,

$$d\hat{n} = \begin{pmatrix} e & f \\ f & g \end{pmatrix} \begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1}; \tag{S43}$$

where E, F, G and e, f, g are the coe cients of the rst (I) and second (II) fundamental forms respectively in the basis $\{\hat{W}, \hat{L}\}$. For example,

$$I(\hat{W}) = \hat{W} \cdot \hat{W}; \tag{S44}$$

$$II(\hat{W}) = \hat{W} \cdot d\hat{n}(\hat{W}). \tag{S45}$$

These coe cients are given by,

$$E = \hat{W} \cdot \hat{W} = 1; \quad F = \hat{W} \cdot \hat{L} = \cos\theta_1; \quad G = \hat{L} \cdot \hat{L} = 1;$$
$$e = \frac{d\hat{W}}{dW} \cdot \hat{n} = \kappa_W; \quad f = \frac{d\hat{W}}{dL} \cdot \hat{n} = \frac{d\hat{L}}{dW} \cdot \hat{n} = 0; \quad g = \frac{d\hat{L}}{dL} \cdot \hat{n} = \kappa_L.$$

By substituting the above expressions we get,

$$d\hat{n} = \frac{1}{\sin^2\theta_1} \begin{pmatrix} \kappa_W & -\kappa_W \cos\theta_1 \\ -\kappa_L \cos\theta_1 & \kappa_L \end{pmatrix}; \tag{S46}$$

with the eigenvalues or the principal curvatures given by,

$$\kappa_p = \frac{(\kappa_W + \kappa_L) \pm \sqrt{(\kappa_W + \kappa_L)^2 - 4\kappa_W \kappa_L \sin^2\theta_1}}{2\sin^2\theta_1}. \tag{S47}$$

For an orthogonal lattice with $\theta_1 = 90^\circ$, it can be seen from the above expression that the principal curvatures are κ_W, κ_L. The principal curvatures of the bent con guration are given by the eigenvalues of the matrix $d\hat{n}$. The negative of the ratio of these principal curvatures gives the PPR in bending (ν^p_{bend}).

IV: Comparison of Poisson's ratios in stretching and bending

In Fig. S5, we compare LPR and PPR for various folded states in stretching and bending. We note that the Poisson's ratio values of Trimorph metamaterial are unbounded [8]. We nd that similar to previous studies involving other origami patterns [4, 5], Trimorph also

FIG. S5. Comparison of analytically calculated Poisson's ratios in stretching and bending, along lattice and principal directions. The insets show the mismatch between stretching and bending for Principal Poisson's Ratio. The results correspond to $= 60$ and $= 10$.

exhibits the unique property that the bending and stretching Poisson's ratios are equal and opposite, when measured along the lattice directions. Although we did not do analytical veri cation for the Trimorph pattern, we have performed extended precision calculations up to 500 decimals to verify the claim. On the other hand, we found that such a relation does not hold true for Poisson's ratios measured along the principal directions.

V: Extended explanation about tristability and multistability

Here we elaborate on how to nd the rest angles of $_1$ and $_2$ to obtain the tristable Trimorph unit cell. Due to the symmetry between $_1$ and $_2$, we know that con guration (2') in Fig. 6A of the main text happens when $_1 = _2$. When $_1 = _2$, we know that $_3 = _4$, and $=$. This special con guration is de ned by:

$$\cos_{3;(2^0)} = \frac{\cot(\ )\sin(\quad) \quad \csc(\ )\sin(\ ) \quad 1}{\cos(\quad) + 1}; \tag{S48}$$

and consequently,

$$\cos_{1;(2^0)} = \frac{\cos_{3;(2^0)}\sin(\quad) + 2\cos \quad \sin}{\sin(\ + \ )}: \tag{S49}$$

Since con gurations (1'), (2'), and (3') are of the same energy level, on Fig. 6A, the three corresponding points are on the same circle of energy contour. The center of this circle

marks the rest angles: $_1$ and $_2$, and we know that $_1 = \ _2$ due to symmetry. The other two stable con gurations (1') and (3') are the solutions of two nonlinear equations. The rst equation is Eq. (S13), which states that (1') and (3') must be on the kinematic path of the Trimorph origami. The second equation is that the distance from (1') or (3') to the center is the same as from (2'), which is already known:

$$(\ _1 \quad _1)^2 + (\ _2 \quad _1)^2 = 2(\ _{1;(2^0)} \quad _1)^2: \tag{S50}$$

Because the center of the energy contour lies on both the line of $_1 = \ _2$ and the line that is normal to the kinematic path, we can solve for it for any point on the kinematic path:

$$_1 = \frac{k_p \ _1 \quad _2}{k_p \quad 1}; \tag{S51}$$

where k_p is the slope of the normal direction of the kinematic path for any point $(_1; \ _2)$ on the path. Let $f(\ _1; \ _2)$ be the left hand side of Eq. (S13), we obtain k_p as:

$$k_p = \frac{@f(\ _1; \ _2)=@_2}{@f(\ _1; \ _2)=@_1}: \tag{S52}$$

Plugging Eq. (S51) into Eq. (S50), and solving the two nonlinear equations numerically, i.e., Eq. (S50) and Eq. (S13), we can obtain the locations of points (1') and (3'), as well as the location of the center, on the $_1$- $_2$ graph.

When we explore the multistable states of the 2D tessellations of the Trimorph origami, we can introduce inhomogeneity by hybridizing the eggbox and Miura modes. It is noted that although the Miura mode unit cells and eggbox mode unit cells can exist compatibly in the same tessellation, their primitive vectors are di erent, as demonstrated in Fig. S6.

FIG. S6. (A-C) Change of primitive vectors between the eggbox and Miura modes. Owing to the middle row of unit cells in Miura mode, the two rows in eggbox mode do not share the same base plane. The white dashed lines indicates the bases (de ned by primitive vectors) on the eggbox mode rows, and the red arrow shows the mismatch between these two bases.

VI: Fabrication of physical prototypes

Three types of triclinic unit cells were designed to create the multistable 2D tessellation, to carry out the Poisson's ratio quantitative experiments, and to realize the 3D metamaterials.

A: Trimorph unit cell, 1D assembly, and 2D tessellation

The 2D (3 3) multistable tessellation was obtained by assembling 9 rigid bimorph origami unit cells. Each unit cell comprises 4 rigid panels jointed together by 4 hinges, 2 elastic and 2 free. Both panels and hinges were designed to mimic as much as possible the ideal unit cell investigated theoretically characterised by the geometrical parameters $a = c = 40$ mm and $= 60$ and $= 10$. Therefore, the panels composing the unit cell have opposite acute interior angles of 50, 70 and 60 . Speci c seats were engraved on the panels to duly housing the hinges and permit the perfect assembly of the unit cell and of the 2D tessellation. The depth of the seats, the actual dimension of panels, and the thickness of the rubber hinges were tailored in a way that, when jointed, the intersection line of the median plane of each panel is very close to the rotation axis of the hinges (maximum tolerance of 1.1 mm), as shown in the sketch of Figs. S7(A) and (B). This is a fundamental condition so that the unit cell, and then the tessellation, exhibit the tristable behavior. In fact, origami-based mechanisms require precision to function as theoretically predicted. For this purpose, a parametric design process was carried out in SolidWorks by varying the main geometrical parameters, e.g. depth of the seats and thickness of the rubber hinges. Each cell panel was milled from a 2 mm thick PC (Makrolon® Polycarbonate, E=2300 MPa, by Bayer) sheet with a Roland EGX-600 CNC 3-axes engraving machine (accuracy 10 m). The free hinges (hinge type I in Fig. S7(C)) were cut from a 1 mm thick white Polypropylene sheet using the same milling machine used for the PC panels. Polypropylene was selected because it has the required folding performance and fatigue resistance. This allowed creating very thin (0.2 mm) folding lines giving rise to zero energy hinges, namely almost perfect free hinges, as depicted in Fig. S7(C). Such folding lines were obtained by a ball end mill (radius 1 mm) mounted on the milling machine. The elastic hinges, shown in Fig. S7(C), were realised by cutting a silicon rubber solid (600 mm 30 mm 40 mm) with a Roland MDX-540 4-axes CNC engraving machine (accuracy 20m), which has rest angle of 62.434 deg (1.0897 rad),

Fig. S7. Front view (A) and back view (B) of the rigid panels composing the triclinic tristable unit cell. In the gure, the details of the seats to duly housing the hinges are also highlighted. (C) Details of the hinges used to build the unit cell (rubber hinges and type I polypropylene hinges), the 1D and 2D tessellation (type II polypropylene hinges). (D) Assembled tristable unit cell. The colored paper disks (10 mm in diameter) indicate the unit cell elements with acute interior angles of 50 deg (orange), 60 deg (green and blue), and 70 deg (red). (E) 1D (3 1) assembly. (F) 2D tessellation (3 3).

as prescribed by the theory. Such rubber hinges lead to the multistable states of the cell as they are able to store energy while folding. Finally, the unit cell was built by bonding together the panels and hinges (Fig. S7(D)). As the panels are characterized by di erent interior angles, di erent colored paper disks (10 mm in diameter) were attached to their faces to interpret the unit cell's elements, as shown in Fig. S7(C). This helps also to visualize, at rst glance, which panels are being activated during the snapping mechanism. The orange, green, red, and blue disks indicate, respectively, the panels with interior acute angles of 50 deg, 60 deg, 70 deg, and 60 deg. A drawback of using rubber and Polypropylene components is their quite di culty bonding. Speci c instant adhesives were used to circumvent this problem. First, the rubber and Polypropylene surfaces were prepared with a Loctite SF 770 (by Henkel) primer. This product is speci c to improve the adhesion of polyole n