

Flutter instability in elastic structures

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Abstract. Flutter instability caused by follower loads has become a reality after the invention of the "freely-rotating wheel device" by Bigoni and Noselli, of the "flutter machine", and of the device to generate Reut-type loads. Further research has proven that flutter instability, Hopf bifurcation, dissipation instabilities, and the Ziegler paradox are all possible in conservative systems, thus disproving an erroneous belief continuing since at least 50 years. Finally, a new type of flutter instability has been addressed, generated by the "fusion" of two structures which are separately stable, but become unstable when joined together. The analysis of instability involves here the treatment of a discontinuity in the curvature of a constraint.

Introduction

Flutter instability is a dynamic behaviour consisting in a blowing-up oscillatory motion, a phenomenon discovered in structural mechanics almost a century ago because of the application of non-conservative (follower forces) to structures. Beside aeroelastic flutter (which will not be considered here), research in this field embraces mechanobiology, growing of plant shoots, motility of cells through eukaryotic cilia, vertebral segmentation in embryos, control problems, the design of soft robotic actuators, graphene peeling, wire drawing, the deployment and retrieval of space tether systems, and solar sails. In the following, new findings are presented in this research topic, including experimental methodologies to follower forces [1-4], non-holonomic constraint [5], effects related to non-smoothness of the equations of motion [6].

The flutter machine

The concept of a follower force is controversial, because the application of these kind of forces to elastic structures leads to several unexpected and counter-intuitive effects: (i.) the lack of Eulerian instability; (ii.) the presence of a Hopf bifurcation; (iii.) the destabilizing effect of dissipation; (iv.) the so-called 'Ziegler paradox' [7]. In this context, Koiter [8] pointed out that follower forces should have been considered as mathematical abstractions, not reproducible in laboratories. As a consequence of that, several attempts to generate these forces have been criticized. A new approach was proposed by Bigoni and Noselli [1] and later perfected with the so-called 'flutter machine' by Bigoni et al. [2, 3], in which a tangentially follower force is generated through the sliding with friction of a freely rotating wheel against a moving support, Fig. 1.



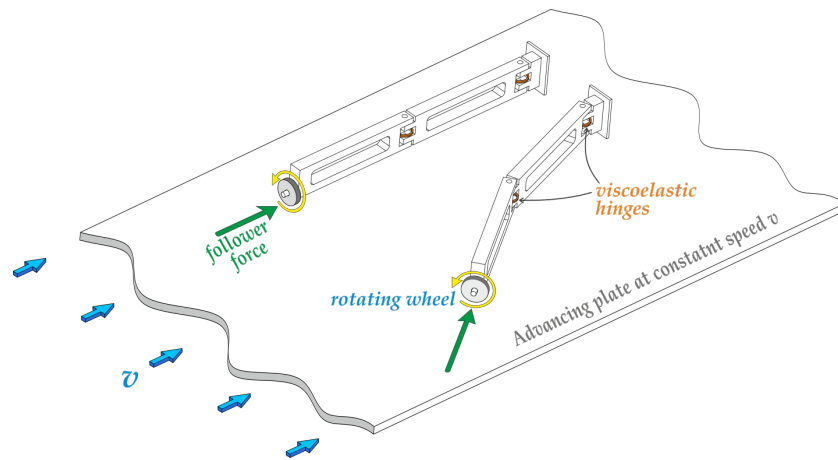


Fig. 1: The way to generate a follower force through the sliding of a freely-rotating wheel against a movable constraint. The wheel is mounted at the free end of a Ziegler double pendulum.

The new experimental set-up has permitted for the first time the systematic validation of the model of following forces and has led to a confirmation of the above listed features (i.)-(iv.). The concepts developed for follower forces also suggested the design of a device to produce forces of the Reut type [4].

Non-holonomic constraints

The concept of the sliding wheel has prompted the idea of substituting the wheel with a similar but purely non-holonomic constraint, Fig. 2. These constraints provide a prescription on the velocity of the end of structure, but not on its position.

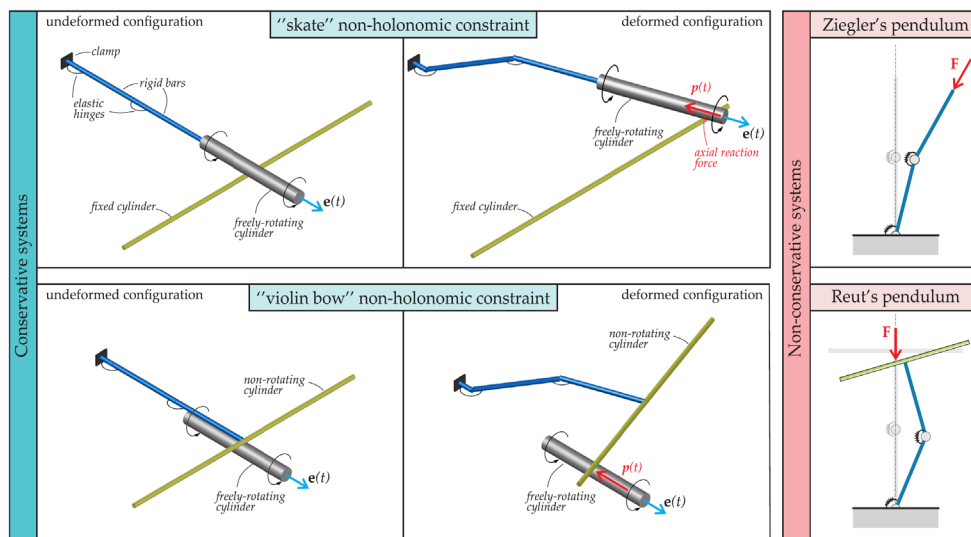


Fig. 2: Application of non-holonomic constraints to generate flutter instability in a double pendulum through the generation of forces similar to those pertinent to the Ziegler's and the Reut's structures.

The remarkable feature emerging from the analysis of non-holonomic constraint is that the latter preserves conservation of energy. Therefore, it is found that flutter instability, Hopf bifurcation, the destabilizing effects of viscosity, and the Ziegler paradox are all possible even within a conservative framework.

Non-smoothness of the equations of motion

As a continuation of the above-presented studies, an elastic structure subject to a follower force was considered in which one end can slide against a linear spring along a smooth constraint presenting a discontinuity in the curvature, as shown in Fig. 3.

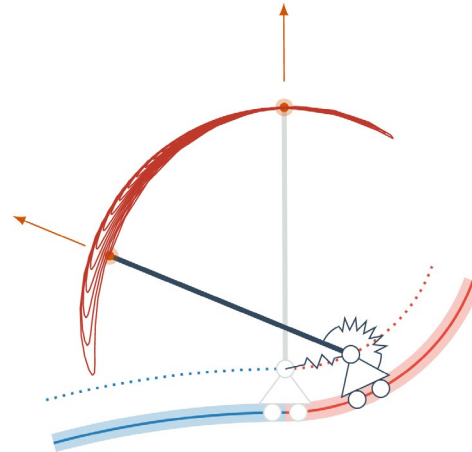


Fig. 3: A structure governed by a non-smooth system of differential equations showing flutter instability as a consequence of the application of a follower force together with a constraint with a jump in the curvature.

The dynamics of the structure is governed by a non-smooth system of differential equations leading to a flutter instability. The most relevant point is that such system can be viewed as the ‘fusion’ of two different structures, each with a circular sliding profile. Interestingly, the two separate structures are stable under the same load for which the compound structure is unstable. Therefore, the mechanical instability is the effect of both the follower nature of the load and of the discontinuity of the curvature of the sliding profile.

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