Experiments on fracture trajectories in brittle materials with voids

D. Misseroni(1), D. Bigoni(2), A. B. Movchan(1) and N. B. Movchan(1)

(1) Department of Mathematical Sciences - University of Liverpool, UK

(2) Department of Civil, Environmental and Mechanical Engineering - University of Trento, I

Email: diego.misseroni@unitn.it

Web: http://www.ing.unitn.it/~diego.misseroni/index.php

Summary

Systematic experiments and computational simulations were performed to investigate the validity of an asymptotic model to predict crack trajectories in brittle materials containing isolated voids. The experiments were performed by the quasi-static loading of v-shaped notched plates of brittle material under Mode-I. We have also identified both large holes and the dynamic regime where fracture surfaces show kinking and roughness as the limitations of the considered models.

Keywords

Asymptotic elasticity, crack growth, fracture mechanics, X-FEM.

Introduction

In the case of brittle materials such as advanced ceramics, rock and glass the prediction of fracture patterns has many implications in a wide range of contexts. Crack propagation when triggered by external loads can have catastrophic consequences and theories that can predict the geometrical shape of cracks are rare. Moreover, crack trajectory is influenced by the presence of defects, such as voids and inclusions. These considerations show clearly the importance of having an analytical model to describe the fracture mechanism. Crack branching and path deflection in two-dimensional elastic solids were examined by Sumi et al. (1983) [9] and computational models have been developed by Xu et al. (1994; 1998) [11][12] to study the crack growth in heterogeneous solids. Formulations for a curved crack based on a perturbation procedure were discussed by Hori and Vaikuntan (1997)[2], although no weight functions were involved in their analysis. A two-dimensional asymptotic model (both plane stress and plane strain) for the determination of the crack trajectory interacting with elliptical inclusion has been developed in the analytical form by Movchan and his co-workers (see [1], [5], [7], [3] and [4]). Their model assumes a semi infinite crack growing quasi statically in an infinite, brittle, isotropic and linear elastic body under pure Mode-I loading (K_{II}=0) and interacting with isolated defects ‘far’ from the straight trajectory that would be followed by the crack in the absence of disturbances. In the case where more than one defect is present, we assume they do not interact with each other (Fig. 1). The elastic properties of the matrix and the elliptical defects are defined via the Lamé constants.

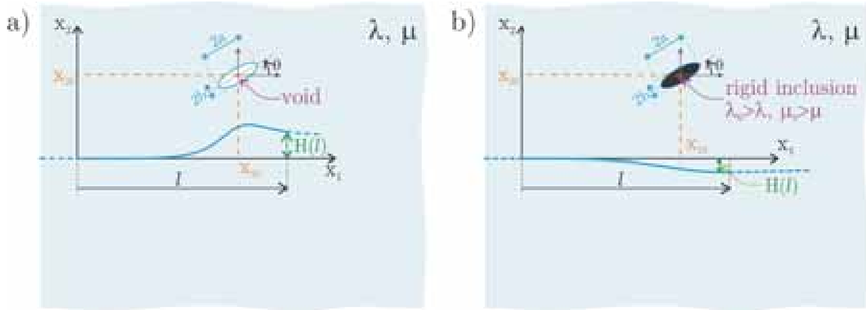

Fig. 1: The analyzed crack geometry interacting with elliptical defects.

The crack path was predicted using elementary functions or straightforward integrations involving weight functions and the voids were characterized by their dipole tensors [8]. In the case of plane stress and elliptical void the crack trajectory can be described by the follow closed-form formula (see[10]):

$$H(l) = \frac{(1-\nu^2)R^2}{2x_2^0}\Big[2(1+m^2) - t\Big(2 + t - t^2 + m^2(1+t)$$
$$+ 2m\cos 2\theta(1+2t)(1-t^2) - 2m\sin 2\theta(2t-1)(1+t)\sqrt{1-t^2}\Big)\Big], \quad (1)$$

where: $R = \dfrac{a+b}{2}$ and $m = \dfrac{a-b}{a+b}$.

Adaptive numerical simulation and experiments

The above asymptotic model to predict quasi-static crack growth in brittle material with voids was verified via both computational techniques and real experiments. The extended finite element method (X-FEM) to simulate crack advance without any remeshing rules was employed together with a parametric python script that is run by means of MatLab. The damage evolution was modelled using a Linear Elastic Fracture Mechanics approach (LEFM) specific for brittle fractures. The strain energy release rate at the crack tip was calculated using the virtual crack closure technique (VCCT). We simulated mix-mode behavior (tolerance=0.1, viscosity=0.0001) by the BK law and we have assumed the criterium of the Maximum Tangential Stress (MTS) to select the normal direction for the crack plane. The computational and experimental set-ups were chosen to be identical in terms of sample size, method of load application and material properties. The load was applied by imposing the grips' displacement at 0.8 μm/s, in order to maintain a low loading rate (quasi-static condition).

The experimental verification of the theoretical asymptotic model was performed (at the 'Instabilities Lab', http://www.ing.unitn.it/dims/ssmg/) by quasi-statically loading (controlled displacements) notched plates of a brittle material under Mode-I conditions until failure.
For the study of crack propagation we made V-shaped notched samples (white and black 2099 Makrolon UV from Bayer, with elastic modulus 2350 MPa, ν=0.35) by cutting a PMMA plate with a EGX-600 Engraving Machine (by Roland), in agreement with the standard test method ASTM

E647-00. The notches were cut to a fixed depth of 0.4 mm in order to trigger rectilinear crack propagation, as shown in the inset in Fig. 2 (a). Two samples of different dimensions were used, namely, 125 mm x 95.0 mm x 3.0 mm and 130 mm x 105 mm x 3mm, for testing one or more circular or elliptical voids, respectively. The experiment was set up to ensure the required quasi-static growth of a crack. The post-mortem crack trajectories and the numerical modelling were then analyzed and compared to the predictions of the asymptotic method as reported in Fig. 2.
Theoretical results, experimental results and numerical simulations are in high agreement when the hypothesis on which the asymptotic model is based are respected.

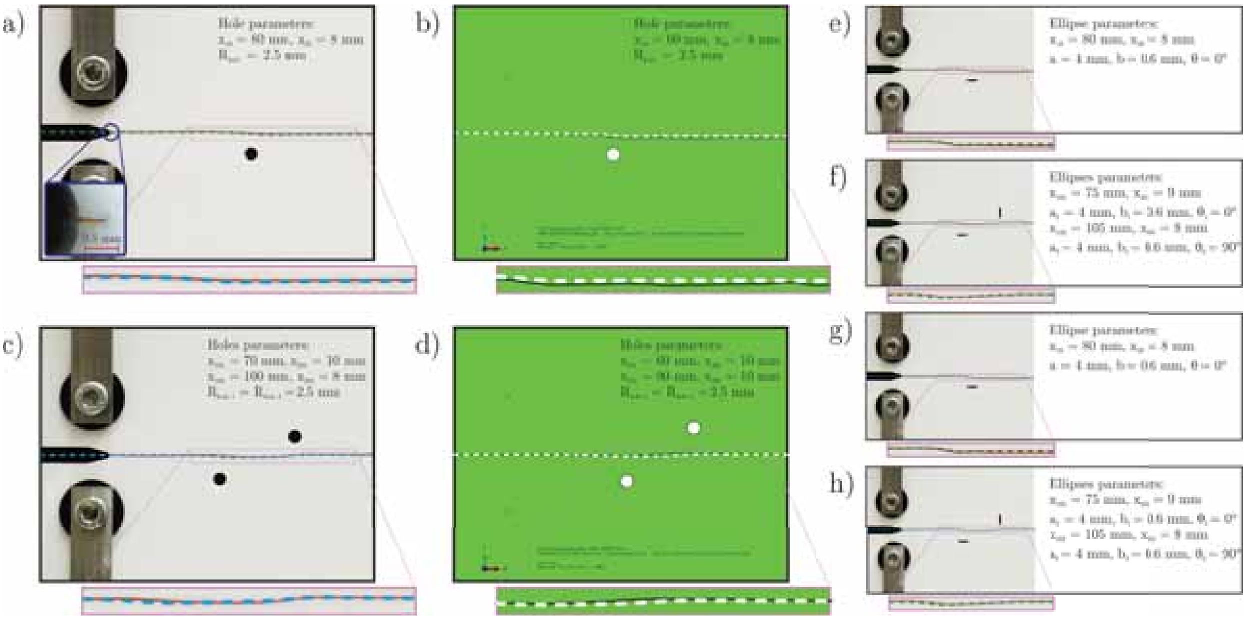

Fig. 2: A comparison between experimental crack trajectories (on the left and on the right) and X-FEM Abaqus simulations (in the middle) with asymptotic formulations (reported with a dashed line) for different cases: one (a, b) and two (c, d) circular voids and one (e, g) and two (f, h) elliptical voids. The parameters indicated in the figure are explained in Fig. 1.

We have highlighted the limitations of the asymptotic approach to the cases of quasi-static crack advance, showing that in the dynamic regime an instability may occur, leading to crack kinking. We have also tested several geometries of small voids at different orientations.

Conclusions

Using both experimental and computational tecniques we have investigated both the applicability and the limitations of an asymptotic model developed to predict the path of a crack propagating in a brittle material. From the experiments, we found that the relative size of the voids that interact with the crack and a dynamic instability leading to the kinking of the crack path are the main limits of this model. In fact, as soon as the ratio between the size of the voids and the distance from the unperturbed crack trajectory and the center of the voids increases, the approximation error of the asymptotic model also increases. In conclusion we can claim that when the underlying hypotheses remain valid, the asymptotic model gives excellent results. This work is important because it allows for the accurate prediction of crack trajectories that interact with inhomogeneities in the form of small inclusions or voids, and could be particularly useful in the design of ceramic articles containing small defects.

Acknowledgments

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