EXPERIMENTAL PROOF FOR THE CLOAKING OF FLEXURAL VIBRATIONS IN A STRUCTURED PLATE

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ABSTRACT

An elastic invisibility cloak embedded in a mechanical lattice is designed, realized and validated by means of both FE simulations and physical experiments. Experimental set-up and numerical simulations consider three different lattices, namely a homogenous lattice, a lattice with a hole, and a lattice with a cloaked hole. The qualitative assessment of the efficiency of the cloak is provided experimentally by using the Hooke-Chladni-Faraday technique. The elastic lattice cloak, implemented experimentally, shows high efficiency.

1 INTRODUCTION

The present work shows the scattering reduction of flexural waves propagating in a structured plate, as reported in Fig. 1. The cloak design is assumed in agreement with the theoretical findings reported in [1] for membrane waves and in [2] for flexural waves in plates.

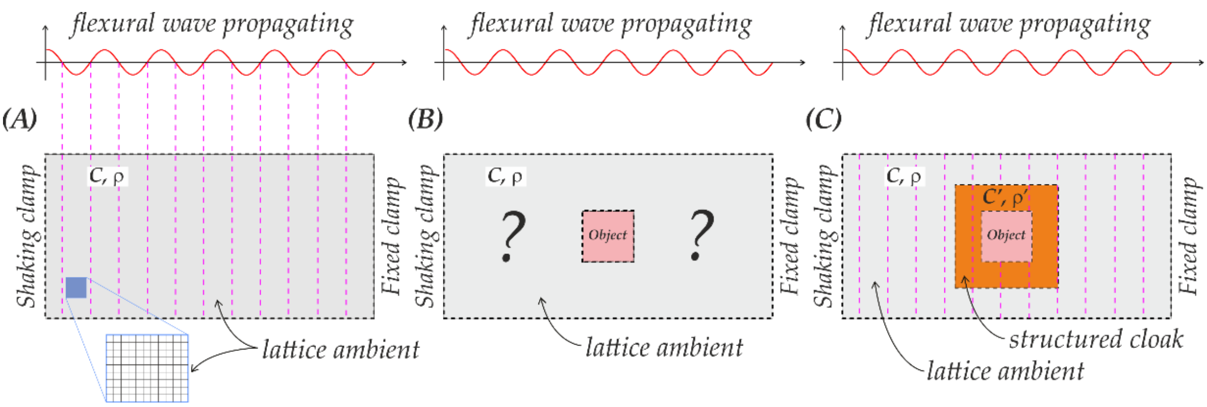

Figure 1: The three different lattices analysed numerically and experimentally to verify the efficacy of the structured square cloak in reducing the scattering of flexural waves by a void. (A) An homogeneous lattice, (B) a lattice with a void, (C) a lattice with a void surrounded by a structured cloak. In the case (B), a scatter of the wavefront is observed because of the presence of the void. In the case of the presence of the cloak (C), the wavefront is restored as the void was absent.

Unlike the majority of the published works on this field which are based on the so-called "cloaking transformation" [3, 4, 5, 6], we derive the mechanical and geometrical properties of our coating using the "regularized", namely non-singular, cloaking transformation [1, 2].
The first experimental implementations of multi-scale mechanical cloaks were performed by the group led by Wegener [7, 8, 9]. The cloak proposed by Wegener's group is embedded in a continuum, so that its practical realization requires an intermediate step of approximation [7]. By contrast, our novel design is embedded in a structured medium and is appealing for its simplicity and ease of realization.

The cloaking action is shown in the form of restored wavefronts behind a square hole surrounded by a lattice cloak.

2 THE THEORETICAL MODEL: THE NON-SINGULAR CLOAK

The geometrical properties of the structured near-cloak presented here are the implementation of the theoretical framework reported in [2] that is based on the use of a regularized push-out transformation. The main idea is to map a domain with a small hole (e.g. a square of semi-width $a\varepsilon$) into another domain, under the constraint that the exterior boundary is preserved while the interior boundary is expanded to the required finite size (Fig. 2A).

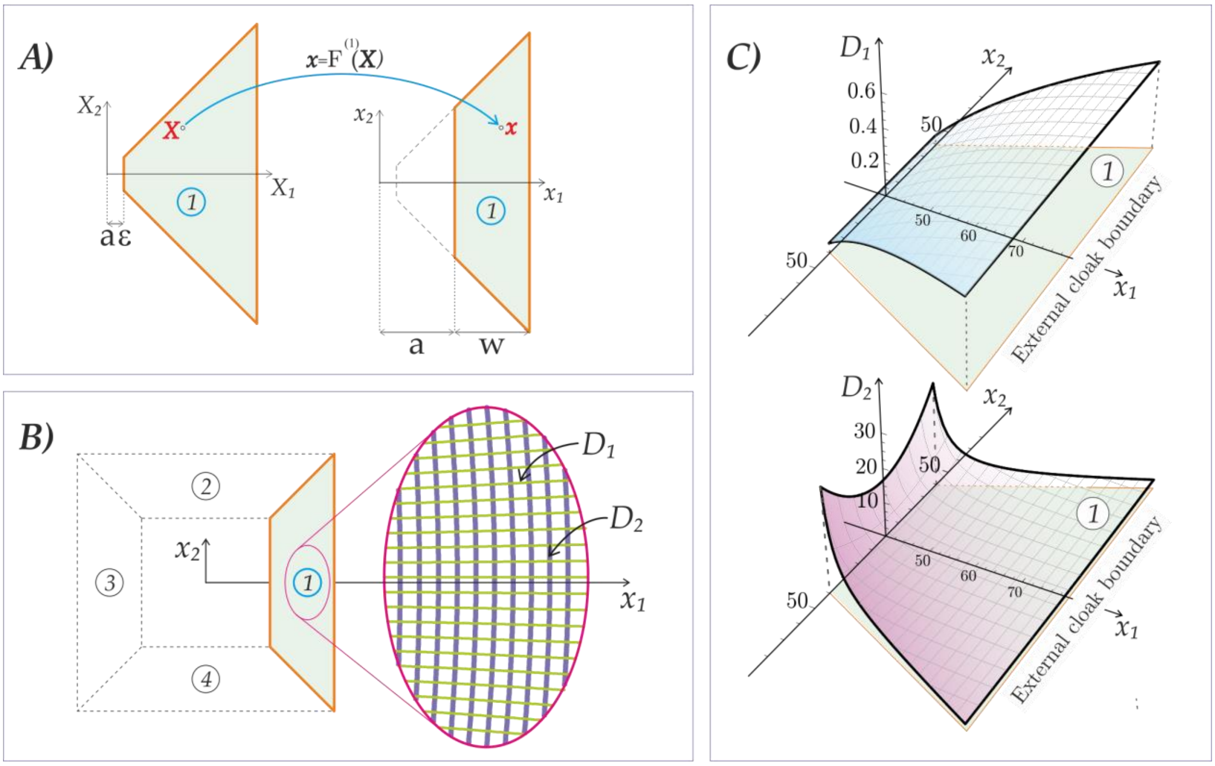

Figure 2: (A) A representation of the push-out transformation: the function $F^{(1)}$ maps the undeformed trapezoidal region (1) to the deformed trapezoidal configuration. (B) A scheme of the discrete lattice structure where the curved ligaments are oriented following the principal directions of the stiffness matrix for the continuum cloak. (C) The required stiffnesses D_1 and D_2 for the cloak ligaments reported as function of x_1 and x_2.

The approximate cloak is obtained with curved ligaments aligned with the principal directions of the stiffness matrix for the continuum cloak, as illustrated in Fig. 2B. The ligaments have rectangular cross section and may be treated as beams characterised by the bending stiffnesses D_1 and D_2 chosen in accordance with the graphs of Fig. 2C. The structured cloak is then installed around the void present in the ambient square lattice which is subjected to out-of-plane flexural vibrations.

3 NUMERICAL AND EXPERIMENTAL RESULTS

Both experiments and three-dimensional finite element (FE) computations, performed with ABAQUS, were carried on to verify the efficacy of the structured cloak in reducing the scattering of flexural waves by a void. We show that for frequencies lower than 270 Hz, the cloak reduces significantly the scattered field. We compare the scattering wave front for three different cases: the first for a homogeneous lattice in the absence of any void; the second in the presence of a void; and the third in the presence of a void surrounded by our specially designed invisibility cloak. The lattices were constrained by clamps and excited by using a shaker connected to the left clamp. The other two sides of the lattice are free. The boundary conditions were chosen consistently, both in the numerical simulations, and in the physical experiments. The numerical results are reported in Fig. 4A. The actual three structured flexural systems were produced by drilling polycarbonate plates (white 2099 Makrolon UV from Bayer) with an EGX-600 Engraving Machine (accuracy 0.01mm, manufactured by Roland). The mechanical properties of the polycarbonate, namely elastic modulus, mass density and Poisson's ratio, are $E = 2350$ MPa, $\rho = 1200$ kg/m^3 and $\nu = 0.35$ respectively. The experimental set-up employed in the experiments is reported in Fig. 3.

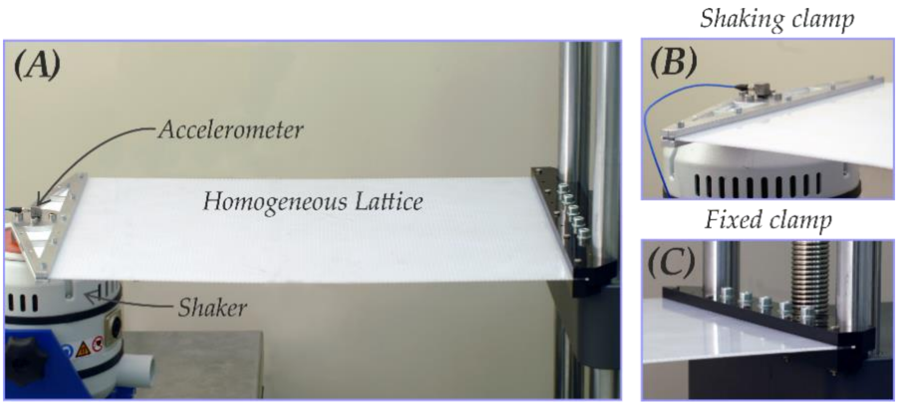

Figure 3: (A) The experimental setup employed in the cymatic experiments. A thin transparent film was used to cover the flexural lattice system to enable the use of powder for the Hooke-Chladni-Faraday visualization; (B) and (C) The detail of the shaking clamp and the fixed clamp respectively.

The qualitative assessment of the efficiency of the cloak was provided experimentally by using the Hooke-Chladni-Faraday technique. This technique shows the positions of the nodal lines of a plate under vibrations.

The quantitative data used for evaluating the effectiveness of the cloak were presented as the set of Fourier coefficients C_k, measured for the scattered fields on a circle of sufficiently large radius, centred around the uncloaked and cloaked holes. These coefficients are compared with the reference Fourier coefficients corresponding to a plate with a small square hole. The results are reported in Fig. 4B. We show that the moduli of the Fourier coefficients in the representation of the scattered field around an uncloaked finite hole are larger than the moduli of the Fourier coefficients for the scattered fields around a cloaked hole of size a and around a hole of a small size εa.

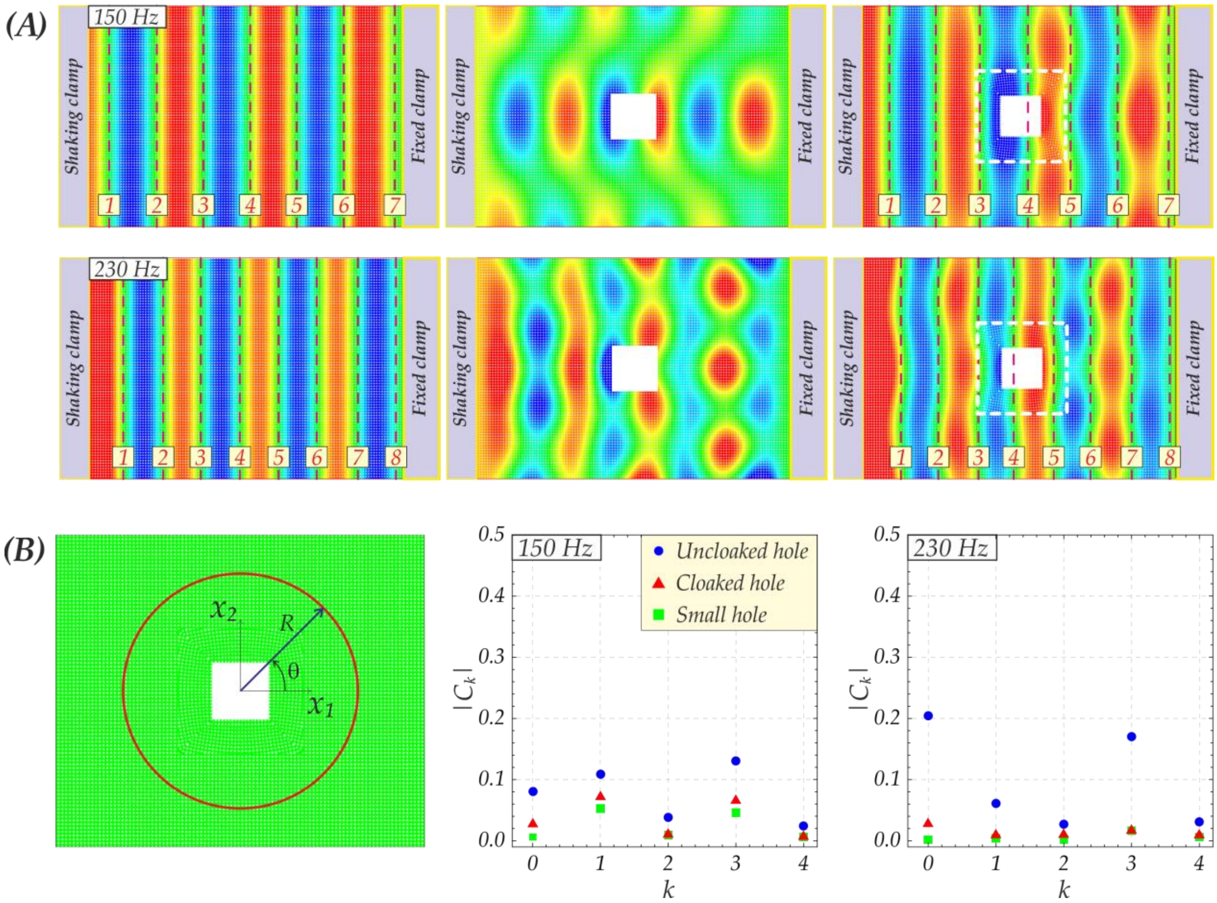

Figure 4: (A) ABAQUS simulations performed in the case of a plate without a hole (left), a plate with an uncloaked hole (middle), and a plate with a hole surrounded by the cloak (right) in the case of an applied displacement with a frequency of 150 Hz and 230 Hz. In the subfigures the dashed/black lines show the positions of the nodal lines of the vibrating plate. (B) A quantitative assessment of the efficiency of the cloak action for the case of 150 Hz and 230 Hz is provided as a set of Fourier coefficient C_k.

The results for the case of oscillatory boundary displacement condition with a frequency of 120 Hz, and reported in Fig. 5 (numerical simulations, upper part, and experimental tests, lower part), confirm that the considered elastic lattice cloak shows high efficiency for frequencies lower than 270 Hz. The results demonstrate the efficiency of the invisibility cloak comparing a uniform vibrating plate without a hole, a plate with an uncloaked hole, and a plate with a hole surrounded by the cloak.

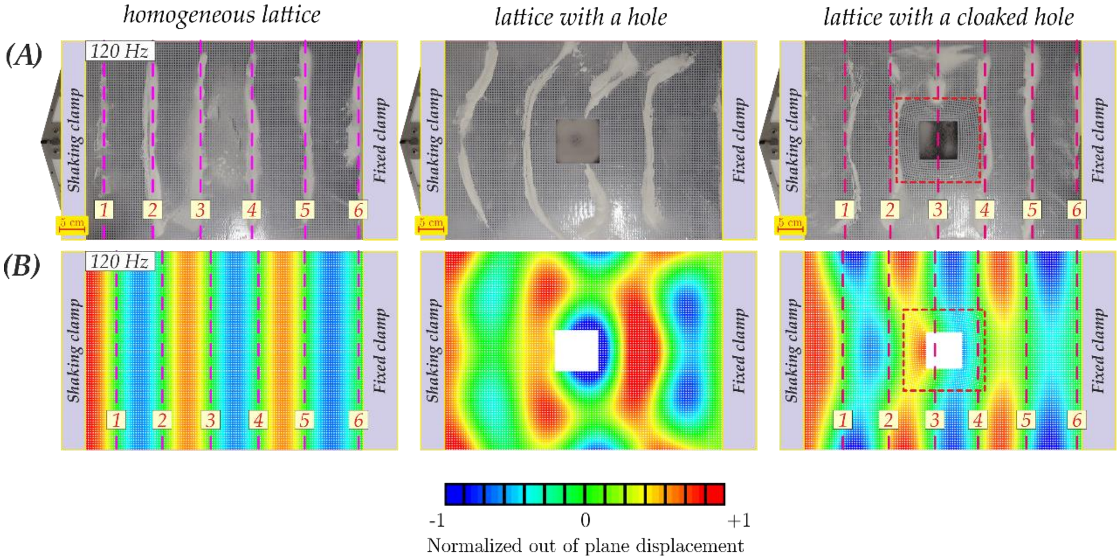

Figure 5: Comparison between the experiments (A) and the numerical simulations (B) in the case of an applied displacement with a frequency of 120 Hz. In the subfigures the dashed/purple lines show the positions of the nodal lines of the vibrating plate. The cloak is highlighted by a dashed/red square.

4 CONCLUSIONS

We have implemented high-precision fabrication and experimental testing of an elastic invisibility cloak for flexural waves in a mechanical lattice. We have proved the efficiency of the cloaking action both numerically and experimentally. The approximate cloak presented here proves to be efficient for frequencies lower than 270 Hz.

ACKNOWLEDGEMENTS

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