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RIGID INCLUSIONS: STRESS SINGULARITY, INCLUSION NEUTRALITY AND SHEAR BANDS

Francesco Dal Corso¹, Davide Bigoni¹ and Giovanni Noselli², Diego Misseroni¹, Summer Shahzad¹

¹ University of Trento, Trento, Italy

² SISSA, Trieste, Italy

Abstract: Analytical solutions in elasticity predict singularities of stress fields at the corners/tips of rigid polygonal/linear inclusions, similarly to the case of void inclusions. On the other hand, a rigid line inclusion is neutral to homogeneous simple shear since a homogeneous stress state is obtained.

We show that: (i) photoelastic experimental investigations validate the rigid inclusion model and therefore the assumptions about infinite stiffness of the inclusion and its complete adhesion with the matrix phase; (ii) when perturbations are superimposed upon a homogeneous pre-stress state, analytical incremental solutions display localization of deformation at the tips of rigid line inclusions and along the shear band directions, confirming experimental observations in ductile and quasi-brittle materials.

Keywords: stiffener; elasticity; failure mechanisms; anti-crack

1 INTRODUCTION

The rigid inclusion model is used to represent inclusions with high stiffness compared to that of the embedding matrix, so that it corresponds to a sort of 'inverse' of a void. Assuming complete adherence to the matrix and the infinite stiffness of the inclusion, the boundary conditions introduced by this model are (differently from a void) of kinematical type, defining the condition that the points of the inclusion can only display a rigid body motion.

Though in the last decades many authors have investigated the theoretical mechanical fields around rigid inclusions, the fundamental task of the validation of this model have been left untouched and it has been only recently investigated [1,2].

On the other hand, a number of instabilities at the micro-scale in form of shear bands have been experimentally observed around thin stiff inclusions in ductile matrixes undergoing large deformation. Application of the perturbative approach to a prestress state close to the ellipticity loss have shown the focussing of deformation in shear bands emerging from the inclusion tips and have disclosed the mechanisms of ductile failure in reinforced materials [3,4,5].

2 VALIDATION OF THE RIGID INCLUSION MODEL

Validation of the rigid inclusion model has been addressed through photoelastic experiments, performed on two-component resin sample containing stiff inclusions of different shapes (thin in [1], polygonal in [2]). The photoelastic fringes obtained with a white circular polariscope are shown in Fig. 1 for the case of a rhombohedral inclusion (left) and a thin inclusion (right) during an uniaxial stress experiment. The photoelastic matrix material has been realized employing a commercial two-part epoxy resin (Translux D180[®] by Axon, with proportion resin:hardener=1:0.95, for the matrix containing the rhombohedral inclusion; Crystal Resins[®] by Gedeo, with proportion resin:hardener=1:1, for the matrix containing the steel lamina). The rhombohedral inclusion and the thin inclusion have been realized using solid polycarbonate and steel lamina, respectively.

The photoelastic fringes are reported in Fig. 1 together with the full-field linear elastic solution obtained with the rigid inclusion model. It can be noted that the linear elastic solution is in a very good quantitative agreement with the photoelastic results, confirming the singular behaviour of the stress fields close to the rigid inclusion tips.

Finally, the stiffener neutrality to uniform shear stress states has been verified (again through photoelastic investigation) on a sample subject to a simple shear deformation parallel to the inclusion, Fig. 2. The strain amount can be detected through the visible distortion assumed by the rectangle (drawn on the sample in the undeformed state).

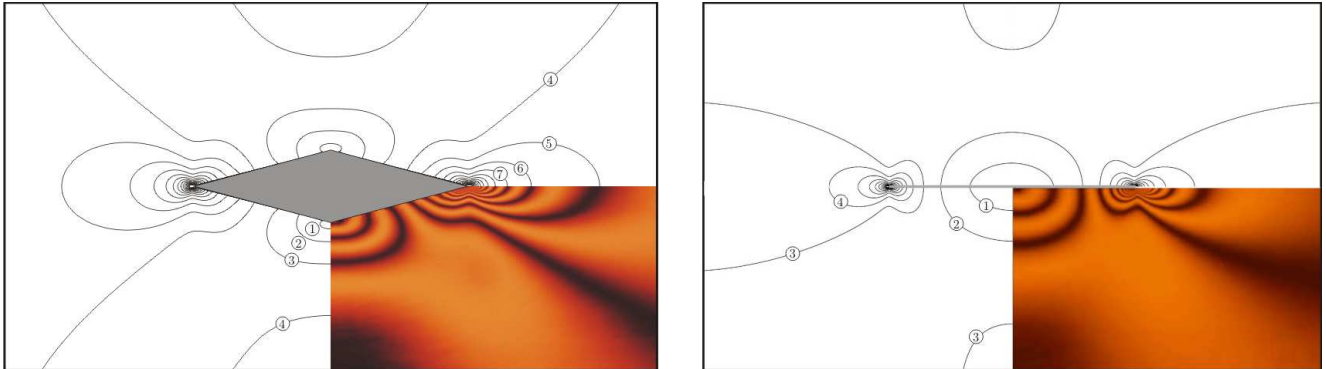


Fig. 1 Photoelastic fringes revealing the stress concentrations around the tips of rhombohedral (left) and thin (right) stiff inclusions during an uniaxial stress experiment.

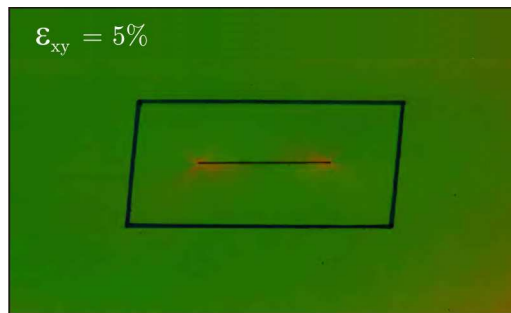


Fig. 2 Photoelastic fringes revealing the stiffener neutrality when the matrix is subjected to a simple shear parallel to the inclusion.

3 THIN RIGID INCLUSIONS PROMOTING FAILURE

Localized deformations in the form of shear bands are experimentally observed to nucleate at the boundaries of stiff inclusions within ductile and quasi-brittle materials.

Assuming the presence of a uniform prestress state, the incremental solutions for uniform Mode I and Mode II perturbations have been obtained for a prestressed material containing a rigid line inclusion [3-5].

Similarly to plane problems for anisotropic materials, the incremental solution is obtained by means of a stream function of complex variables. In particular, the full-field problem is found to be equivalent to a Riemann–Hilbert problem, so that the solutions display square-root singularities at the tips of the rigid inclusion.

Furthermore, since the incremental solution is affected by the prestress amount, exploitation of the solution for prestress states close to the loss of ellipticity shows clearly shear band nucleation and growth at the stiffener tip, Fig. 3, at varying of inclination of the inclusion line with respect to the axes of prestress. The incremental fields are reported for the case of an inclusion embedded in a J_2 -deformation theory material with a prestress amount close to the loss of ellipticity condition. In the case of axes of prestress parallel to the inclusion line (left) the symmetry leads to a symmetric nucleation of shear bands, while in the case of axes of prestress not parallel to the inclusion line (right) the shear band closest to the inclusion line is privileged.

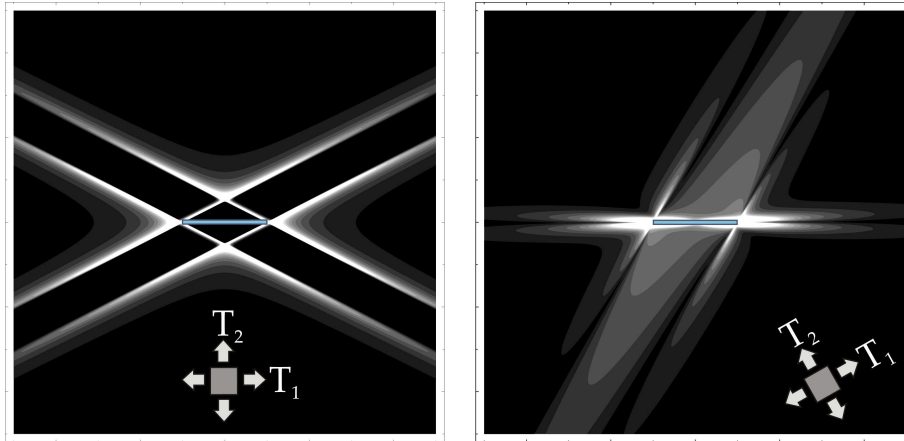


Fig. 3 Level sets of the second invariant of deviatoric incremental strain near a rigid line inclusion for a uniform Mode I perturbation.

4 CONCLUSIONS

The rigid inclusion model has been fully validated by means of photoelastic tests, confirming stress concentration close the inclusion tips of stiff inclusions under Mode I loading and neutrality of rigid line inclusions under Mode II loading, according to analytical solutions in linear elasticity.

The analytical solutions, obtained for the incremental problem of a prestressed material containing a rigid line inclusion, disclose shear band nucleation and growth at the inclusion tips with varying of inclination at varying of the angle between the rigid inclusion line and axes of prestress.

5 ACKNOWLEDGEMENTS

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