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Experiments on fracture trajectories in ceramic samples with voids

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Abstract

Experimental validation is provided for a linear elastic model describing the trajectory of a crack which propagates under Mode I conditions in a ceramic sample, as influenced by non interacting voids. A wide range of experiments were performed by quasi-static loading standard notched unglazed ceramic samples under pure Mode-I loading conditions. It is found that the predicted crack trajectories are in close agreement with the experimental results, so that the model is fully validated and therefore permits correct simulations of crack paths in brittle materials containing small voids.

Keywords: Ceramic products, asymptotic elasticity, crack growth, fracture mechanics

1 Introduction

Ceramic products find a wide range of applications in many contexts: they are used for traditional items, such as plates and tiles, and for high tech components, such as rocket exhaust cones and coatings for windshield glass in airplanes. Moreover, ceramic materials can be formed to provide extraordinary combinations of mechanical, electrical, thermal and chemical properties for enhanced system engineering. Still, the major limiting factor for these materials is the high brittleness, so that failure through crack propagation limits the material usability and can also yield catastrophic consequences. Theories that predict the geometrical shape of crack trajectories during propagation are important since fracture tortuosity is linked to toughness enhancement, but are rare and often difficult to interpret and use in an industrial environment, so that analytical models to describe fracture mechanisms in ceramics are highly valuable and useful for the design.

Experiments on fracture mechanics show that the crack trajectory is deeply influenced by the presence of defects (such as voids or inclusions) at both micro and macro scales. A crack that is propagating in a brittle material deflects from a straight line when it approaches an imperfection and, roughly speaking, is 'attracted' by a void and 'repelled' by a rigid inclusion. However, the prediction of the crack trajectory is a complex mechanical problem. Models for the prediction of

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crack deflection in brittle-matrix composites were developed by Lacroix et al. (2002) [1] and by Wang and Shing [2], the latter considering crack propagation in piezoelectric ceramics. A discussion about the influence of the orientation of the defects with respect to the crack line may be useful to define the most efficient configuration for obtaining the largest deviation of the crack trajectory. In particular, Tsukrov and Kachanov (1997) [3] and Radi (2011) [4] studied the interaction between two holes remotely loaded and noticed that the interaction effects and the energy release rate is maximal not in the ideally symmetric arrangements but in the configurations where the symmetry is perturbed.

The fracture trajectory in two-dimensional elastic solids was examined by Sumi et al. (1983) [5] and computational models were developed by Xu et al. (1994; 1998)[6][7] to study the crack growth in heterogeneous solids. Formulations for a curved crack based on a perturbation procedure in which weight functions were not employed, were presented by Hori and Vaikuntan (1997)[8].

Movchan and co-workers [9, 10, 11, 12, 13] have developed an analytical, two-dimensional model (valid for both plane stress or plane strain) for the determination of the crack trajectory in a linear elastic, but brittle, material. The model considers a semi-infinite crack growing in an infinite elastic-brittle medium and its interaction with small defects in the form of voids or inclusions. The defects are characterized by dipole tensors and the crack path is predicted in terms of elementary functions or by the integration of quantities involving weight functions. The main advantage of this asymptotic model is its simplicity, so that the formula describing the shape of the deflected crack trajectory can easily be used for a variety of fracture configurations and can be implemented as a design tool in many industrial processes.

An attempt to validate the asymptotic model for the prediction of the crack trajectory in ceramics was made by Bigoni et al. [14] and Valentini et al. [15, 16], but their results were far from exhaustive and only indirect (the three-dimensionality of the stress state induced by a Vickers indenter was not coherent with the hypotheses on the basis of the model). More recently, Misseroni et al. [17] presented sound experimental results providing a nice validation of the crack trajectory model, but all obtained on PMMA notched plates, so that results on ceramics have never presented until now.

The aim of the present article is to complement the previous results on PMMA with a study fully dedicated to ceramic materials. In particular, experiments are presented on notched unglazed ceramic tiles, specifically produced for the tests and in which circular and elliptic voids have been manufactured. The experiments validate the reliability of the asymptotic model to predict crack deflections induced by voids for ceramic materials, so that it can be concluded that the approach can be employed in the industrial designs of ceramics.

2 The asymptotic model for Mode-I crack propagation

The asymptotic model for the description of crack-trajectory is summarized in this section (further details can be found in [10, 12, 15]). A semi-infinite crack is considered growing in an infinite, brittle, isotropic, and linearly elastic body, subject to pure Mode-I loading ($K_I > K_{IC}$), and interacting with a finite number of defects. These, in the form for instance of voids or inclusions, are assumed to be 'sufficiently distant' from the straight trajectory that would be followed by the crack in the absence of disturbances. Only the case of *plane stress* and *voids* of elliptical voids is now presented and the elastic properties of the material are defined by the Lamé constants λ and μ. In the case where more than one defect is present, a non-interaction assumption is introduced, so that the solutions for different defects can be simply superimposed.

The position of the elliptical void, with the major and minor semi-axes denoted by a and b respectively, is defined through: (i.) the coordinates of the center (x_1^0, x_2^0) and (ii.) the inclination θ of the major axis with respect to the x_1-axis. Fig. 1 shows a sketch of the problem set-up, with the adopted nomenclature.

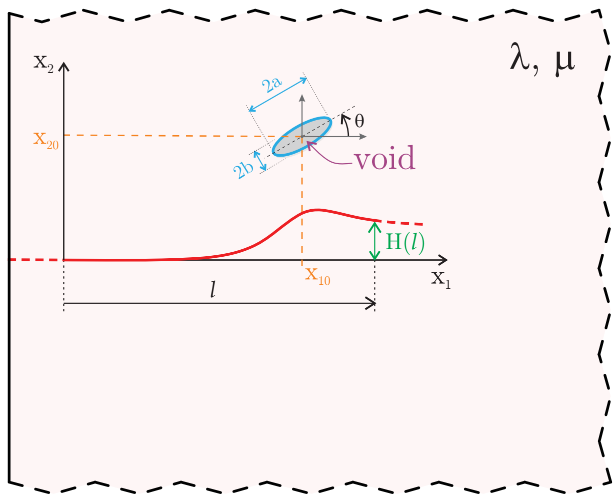

Figure 1: The geometry of a crack interacting with an elliptical void, which is defined by the major and minor semi-axes (a and b respectively), by the coordinates (x_1^0, x_2^0) of the center, and by the inclination θ.

The deflection of the crack trajectory $H(l)$ in the vicinity of the elliptical voids can be described by the following closed-form formula

$$H(l) = \frac{(1-\nu^2)R^2}{2x_2^0}\Bigg[2(1+m^2) - t\Bigg(2+t-t^2+m^2(1+t) + 2m\cos 2\theta(1+2t)(1-t^2) - 2m\sin 2\theta(2t-1)(1+t)\sqrt{1-t^2}\Bigg)\Bigg], \quad (2.1)$$

where l is the crack tip coordinate and

$$R = \frac{a+b}{2} \quad \text{and} \quad m = \frac{a-b}{a+b}.$$

It is important to remark that the expression (2.1) depends on the Poisson's ratio ν, the angle of inclination θ of the major axis, and the parameters R and m of the elliptical void.

The crack deflection at infinity, say, 'far away' from the small void, can be obtained from eq. (2.1) in the limit $l \to \infty$, which for a particular case of an elliptical void becomes

$$H(\infty) = \frac{R^2}{x_2^0}(1+m^2). \quad (2.2)$$

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and does not depend on the orientation of the void.

3 The Experimental validation of the asymptotic model

The experimental validation of the asymptotic model was provided employing the experimental setup reported in Fig. 2. Systematic experiments were performed through quasi-static loading of V-notched ceramic samples under Mode-I conditions.

Notched, unglazed tiles with circular or elliptical defects were specifically manufactured for the experiments (courtesy of SACMI, Italy). Due to their complex shape and the precision needed for the voids it was impossible to use a disk cutter. Instead, all sample were cut using a high pressure water-jet cutter. This system is well-known in the ceramics industry and allows for the cutting of complex shapes with an accuracy of 0.1 mm for the specific width of the tiles employed in the present study. The geometry (width, length, height and grips positions) of the samples prepared for the experiments was in agreement with the standard test method ASTM E647-00, see Fig. 2 (b).

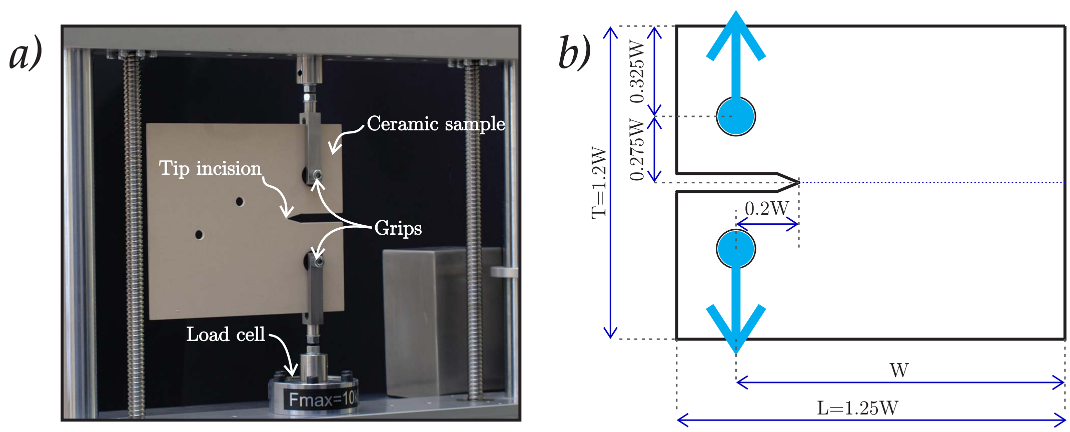

Figure 2: a) The experimental setup employed to provide the pure Mode-I loading on the ceramic samples. b) Sample geometries as prescribed by the standard test method ASTM E647-00

The incisions at the V-shaped notch tip of the ceramic samples are crucial for initiate a straight fracture propagation (after the initiation the fracture curves as a consequence of the presence of the voids) and were realized with a Rockwell C diamond indentor. The indentor tip has a cone angle of 120° and a fillet radius R = 0.2 mm, Fig. 3. The indentor was mounted on a CNC milling machine to scratch the tile at a fixed depth of 40 μm and for a length of 3.5 mm, the latter measured from the external edge of the tiles. The details of the incision procedure is reported in Fig. 3. Samples of two different dimensions were used, namely, 200 mm x 192 mm x 3.0 mm and 240 mm x 210.2 mm x 3.0 mm, for testing circular or elliptical voids, respectively. The position and the size of the voids are shown in the captions of the figures, with reference to the nomenclature introduced in Fig. 1.

The tiles were loaded by imposing displacement with a load frame MIDI 10 (from Messphysik), which provides the pure Mode-I condition. The experiment was set up to ensure the required

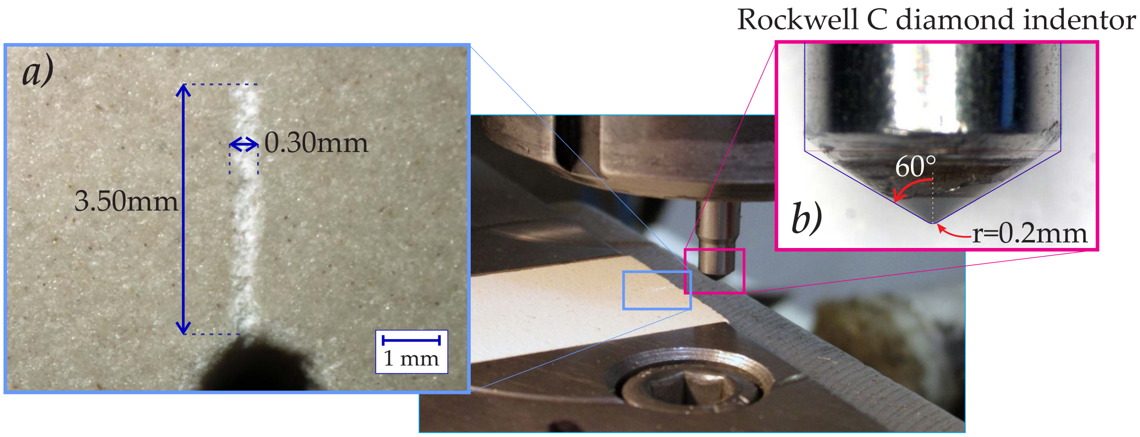

Figure 3: Details of the engraving procedure at crack tip to initiate a straight fracture propagation. The final incision (a) obtained with a Rockwell C diamond indentor (b).

quasi-static growth of a crack, by imposing a low speed of 0.5 μm/s for the load frame crosshead. During the experimental tests the load was measured with an AEP TC4 load cell (RC 10kN). The data were acquired with a NI compactRIO system, interfaced with Labview 2013 (from National Instruments).

Photos were taken with a Nikon D200 digital camera (equipped with a AF-S micro Nikkor lens 105 mm 1:2.8G ED), with a Sony NEX 5N digital camera (equipped with 3.5-5.6/18-55 lens, optical steady shot from Sony Corporation), or with a stereoscopic microscope Nikon SMZ-800 (equipped with objectives P-Plan Apo 0.5X and P-ED Plan 1.5X). The movies were taken with a Sony Handycam HDR-XR550.

All the presented experiments were performed at the 'Instabilities Lab', http://www.ing.unitn.it/dims/ssmg/.

3.1 The comparison between experiments and the mechanical model

Theoretical predictions have been compared with a series of experiments, performed on different samples containing small voids, of circular or elliptical shape, located at different positions and orientations with respect to the notch. The center of each voids was located far enough from the unperturbed crack trajectory to satisfy the assumptions of the model. In other words, the ratio between the voids major dimension and the distance from the crack trajectory that would be followed in absence of a disturbance was selected to be small. Under these conditions, the theoretical predictions of the asymptotic model have been proven to be very close to the experimental measurements.

3.1.1 Crack deflection by the interaction with circular and elliptical voids

Crack trajectories, evaluated through the asymptotic formula, eq. (2.1), have been directly compared to the fracture path observable on the post-mortem samples, obtained through slow loading of the notched ceramic unglazed tiles with circular and elliptical voids. Several experiments have

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been performed for each void configuration in order to provide a statistical proof of the validity of the asymptotical approach. In particular, three different tests have been performed for each void configurations, Figs. 4 and 5.

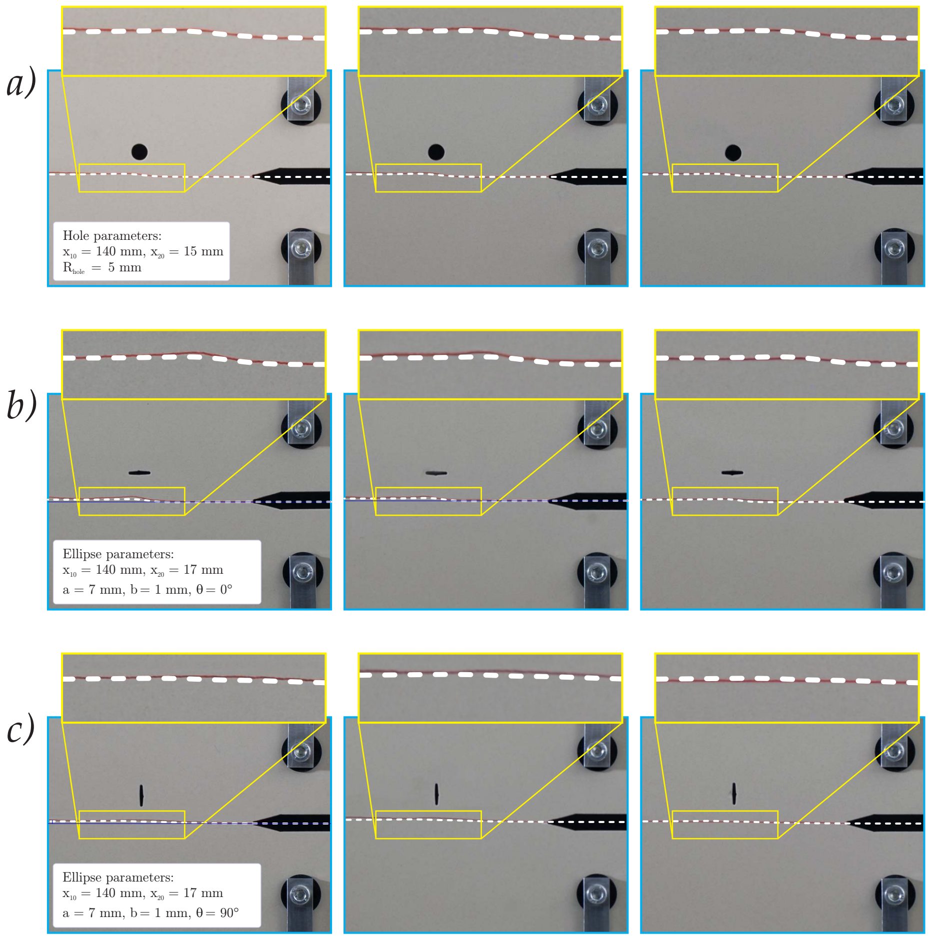

Figure 4: Comparison between experimental crack trajectories, evaluated with photos of post-mortem ceramic samples, and the solution provided by the asymptotic approach. Crack interacting with: one circular void (a), one elliptic void with major axis inclined at an angles $\theta = 0^\circ$ (b) and $\theta = 90^\circ$. Three experiments for each configuration are shown. The theoretical predictions are plotted by a white/dashed line while the crack trajectories are highlighted by means of a solid/red colored strip inserted post-mortem between the two crack surfaces.

Fig. 4 reports the case of a crack interacting with one circular void (a), one elliptical void inclined with respect to the x_1-axis at an angle $\theta = 0^\circ$ (b) and at $\theta = 90^\circ$ (c). The case of two voids (circular (a) and elliptical (b)) interacting with a growing crack are reported in Fig. 5.

The figures clearly show that the asymptotic model matches with high accuracy the experimental results for every configurations of voids.

Another prediction of the asymptotic model that has been verified is that the orientation of an elliptical void does not influence the crack deflation at infinity (in other words, 'far enough'

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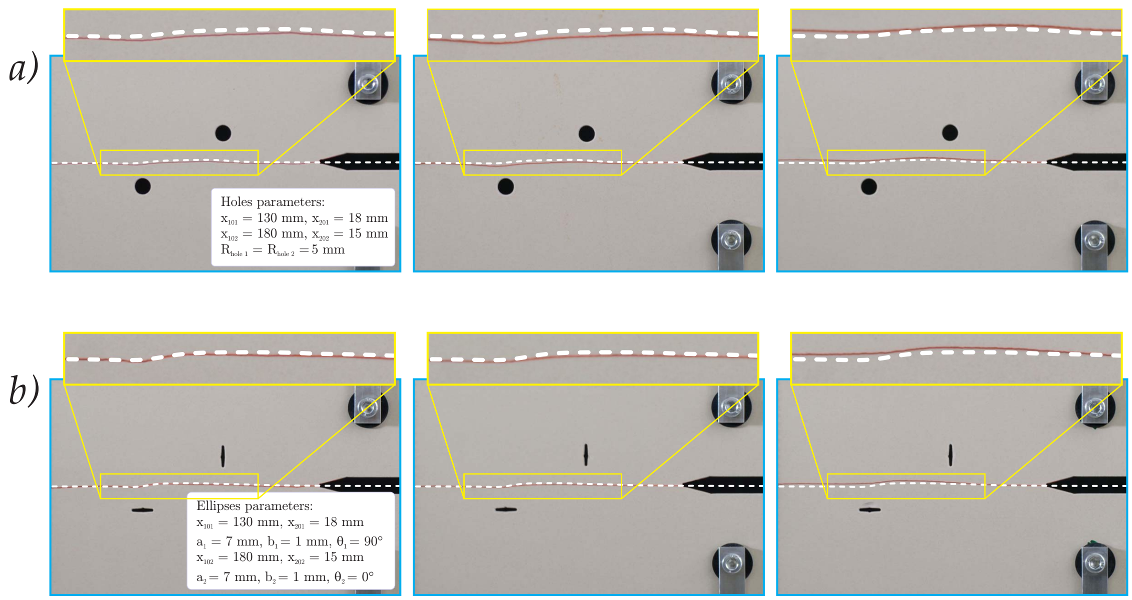

Figure 5: Comparison between experimental crack trajectories, evaluated with photos of post-mortem ceramic samples, and the solutions provided by the asymptotic approach. Crack interacting with: two circular holes (a) and two ellipses inclined at angles $\theta = 0°$ and $\theta = 90°$. Three experiments for each configuration are shown. The theoretical predictions are plotted by a white/dashed line while the crack trajectories are highlighted by means of a solid/red colored strip inserted post-mortem between the two crack surfaces.

from the void) in the horizontal direction. This was done by testing the two limit cases for $\theta = 0°$ and $\theta = 90°$ Fig. 6 shows the deflection $H(l)$, for the both cases, measured close to the external edge of the samples. The comparison between the measured deflections and the deflection $H[\infty]$ predicted from eq. 2.2 are in good agreement. The slight difference between experiments and theoretical predictions is justifiable considering that the the experimental deflection was measured at the external edge of the samples and not at infinity. For this reason it can be concluded that the deflection at infinity is independent of the orientation of the ellipses. Moreover, in the case of two voids (circular or elliptical), the sum of equal but opposite deflections results in a nearly zero total deflection at infinity.

During the experimental tests the applied load was measured. It was found that the failure load of the tested ceramic samples equals about 650-700 N. In Fig. 7 an example of load/displacemet curve is reported as recorded during an experiment on a sample containing an elliptical void inclined at an angle $\theta = 0°$. From the graph the brittle behaviour of the sample can be appreciated, as results from the sudden drop in the applied load.

As a conclusion, the experiments fully validate the theoretical model for a wide range of configurations of voids.

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4 Conclusions

Systematic experiments, involving the interaction of circular and elliptical voids with crack propagation in ceramics, were performed to verify the reliability of an asymptotic model that predicts crack trajectories in brittle materials. The experiments were setup on ad hoc manufactured unglazed tiles to ensure the respect of the underlying assumptions such as crack advance under pure Mode-I loading. The experiments show that the asymptotic model predicts with high accuracy the trajectory of cracks growing in a ceramic material and interacting with some defect. Thanks to its simplicity, the model can be applied in the design of ceramic pieces and used to understand the failure mode of such materials, both traditional and advanced.

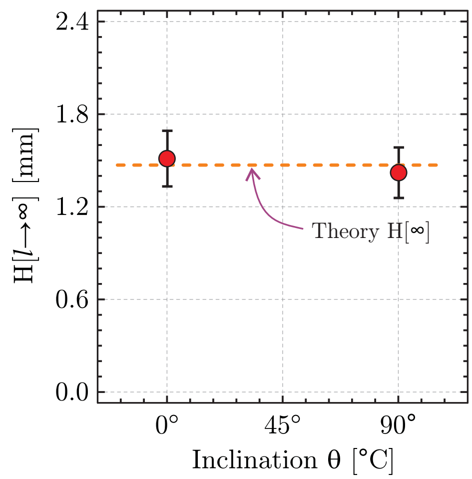

Figure 6: The deflection at infinity predicted by the asymptotic model compared with that post-mortem measured in the experiments. The measurements were taken at the external edge of the samples for two different inclinations of the same elliptical void.

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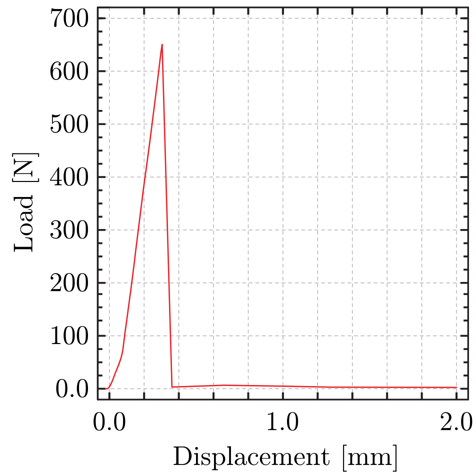

Figure 7: A load/displacement curve obtained during a Mode-I loading of a samples containing an elliptical void inclined at an angle $\theta = 0°$ and using a dynamic load cell BOSE 1010CCH-1K-B

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